Consider the following vector fields:

\[ \mathbf{F}_1 = z \mathbf{k}, \]
\[ \mathbf{F}_2 = x \mathbf{j} + y \mathbf{k}, \]
\[ \mathbf{F}_3 = \frac{x}{(x^2 + y^2 + z^2)^{3/2}} \mathbf{i} + \frac{y}{(x^2 + y^2 + z^2)^{3/2}} \mathbf{j} + \frac{z}{(x^2 + y^2 + z^2)^{3/2}} \mathbf{k}. \]

Let \( S_1 \) be the unit sphere centered at the origin, oriented by the outward normal, and let \( S_2 \) be the sphere of radius 5 centered at the origin, oriented by the outward normal.

1. For each \( \mathbf{F}_i, i = 1, 2, 3 \), do each of the following:
   
   (a) Calculate \( \text{div} \mathbf{F}_i \).
   
   (b) Calculate \( \int_S \mathbf{F}_i \cdot d\mathbf{S} \).
   
   (c) Calculate \( \int_S \mathbf{F}_i \cdot d\mathbf{S} \).

2. Suppose we have a vector field \( \mathbf{F} \) such that, at every point in \( \mathbb{R}^3 \) except possibly \( (0,0,0) \), \( \text{div} \mathbf{F} = 0 \).

   (a) Does it always seem to be the case that \( \int_{S_1} \mathbf{F} \cdot d\mathbf{S} = 0 \)? Explain your answer, using appropriate examples from the previous problem.

   (b) Does it always seem to be the case that \( \int_{S_1} \mathbf{F} \cdot d\mathbf{S} = \int_{S_2} \mathbf{F} \cdot d\mathbf{S} \)? Explain your answer, using appropriate examples from the previous problem.