1. Let $\mathbf{F}$ be a vector field such that:

- The domain of $\mathbf{F}$ is $\{(x, y, z) \mid x^2 + y^2 + z^2 > 2\}$, i.e., $\mathbb{R}^3$ minus the closed ball of radius 2 centered at the origin.
- $\text{curl} \mathbf{F} = \mathbf{0}$ for all points in the domain of $\mathbf{F}$.

Let $C$ be the circle of radius 5 and center $(0, 0, 0)$ in the $xy$-plane in $\mathbb{R}^3$, oriented counterclockwise, as shown above. (The shaded area in the picture represents the points in $\mathbb{R}^3$ where $\mathbf{F}$ is not defined.) Use Stokes’ Theorem to explain, using words and pictures, how we can be sure that

$$\int_C \mathbf{F} \cdot d\mathbf{s} = 0.$$  

2. Let $\mathbf{F}$ be a vector field such that:

- The domain of $\mathbf{F}$ is $\mathbb{R}^3$ minus the $z$-axis.
- $\text{curl} \mathbf{F} = \mathbf{0}$ for all points in the domain of $\mathbf{F}$.

Also, let $C_1$, $C_2$, and $C_3$ be the curves shown below. Note that the $z$-axis, where $\mathbf{F}$ is not defined, is indicated by a bold line.

Finally, suppose that

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{s} = 13.$$  

(continued)
(a) Explain why the fact that $\int_{C_1} \mathbf{F} \cdot d\mathbf{s} = 13$ does not contradict Stokes’ Theorem.

(b) Find the value of $\int_{C_2} \mathbf{F} \cdot d\mathbf{s}$. Briefly explain your answer, using words and pictures.

(c) Find the value of $\int_{C_3} \mathbf{F} \cdot d\mathbf{s}$. Briefly explain your answer, using words and pictures. (Suggestion: Look for an oriented surface whose boundary is equal to $C_1 + C_3$. What does Stokes’ Theorem say in this case?)