SA16
Math 112, Spring 2006

You may find the following definite integrals useful. (I.e., these are given.)

\[
\int_0^{2\pi} \sin^2 x \, dx = \int_0^{2\pi} \cos^2 x \, dx = \pi,
\]

\[
\int_0^{\pi/2} \sin^2 x \, dx = \int_{\pi/2}^{\pi} \sin^2 x \, dx = \int_{\pi}^{3\pi/2} \sin^2 x \, dx = \int_{3\pi/2}^{2\pi} \sin^2 x \, dx = \frac{\pi}{4},
\]

\[
\int_0^{\pi/2} \cos^2 x \, dx = \int_{\pi/2}^{\pi} \cos^2 x \, dx = \int_{\pi}^{3\pi/2} \cos^2 x \, dx = \int_{3\pi/2}^{2\pi} \cos^2 x \, dx = \frac{\pi}{4}.
\]

1. (6.2) 4(a).

2. (6.2) 16.

3. Let \( D \) be the region in \( \mathbb{R}^2 \) given by \( x^2 + y^2 \leq 4 \), \( x \geq 0 \), \( y \geq 0 \). Draw \( D \), and calculate

\[
\iint_D (x^2 + xy) \, dx \, dy
\]

using polar coordinates.

4. Let \( E \) be the cylinder of radius 3 with center the \( z \) axis and \( 0 \leq z \leq 4 \). Draw \( E \), and calculate

\[
\iiint_E (\sin z)(x + y + 2) \, dx \, dy \, dz
\]

using cylindrical coordinates.

5. Let \( E \) be the upper hemisphere of radius 5 centered at the origin. Draw \( E \), and calculate

\[
\iiint_E z(x^2 + y^2) \, dx \, dy \, dz
\]

using spherical coordinates.