Find the solution to the initial value problem:

\[ y'' + 9y = 27, \quad y(0) = 4, \quad y'(0) = 6. \]

**SOLUTION:** First we need to solve the associated homogeneous equation \( y'' + 9y = 0 \). Its characteristic equation is

\[ r^2 + 9 = 0, \]

with roots \( \pm 3i \). Therefore, the general solution to the homogeneous equation is

\[ y_h = C_1 \cos 3t + C_2 \sin 3t. \]

By the method of undetermined coefficients, a particular solution to the given non-homogeneous equation is of the form

\[ y_p = A = \text{const}. \]

Since \( y_p'' = 0 \), we get \( 9y_p = 27 \), i.e., \( y_p = 3 \). Therefore, the general solution to the non-homogeneous equation is

\[ y = 3 + C_1 \cos 3t + C_2 \sin 3t. \]

Since \( y' = -3C_1 \sin 3t + 3C_2 \cos 3t \), from the given initial conditions we obtain

\[
\begin{align*}
3 + C_1 &= 4 \\
3C_2 &= 6.
\end{align*}
\]

It follows that \( C_1 = 1, \ C_2 = 2 \), and the solution is

\[ y = 3 + \cos 3t + 2 \sin 3t. \]