Sample Final Exam

Name:

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1. Consider the following system of linear equations:

\[ \begin{align*}
  x_1 - x_2 &= 4 \\
  3x_1 + kx_2 &= 7.
\end{align*} \]

(a) For which \( k \) does the system have a unique solution?
(b) For which \( k \) is the system inconsistent?
(c) Are there values of \( k \) for which the system has infinitely many solutions?

Solution:
2. Find $a, b, c, \text{ and } d$ so that

$$\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}. $$

Solution:
3. Urn 1 contains 1 white and 4 red balls, and Urn 2 has 3 white and 2 red balls. An urn is chosen at random, with one as likely to be chosen as the other. Then a ball is withdrawn from the chosen urn.

(a) If a red ball is drawn, what is the probability that it came from Urn 2?
(b) If a white ball is drawn, what is the probability that it came from Urn 1?

Solution:
4. In a group of $n$ people ($n \leq 100$), each person is asked to select a number between 1 and 100, write a number on a slip of paper, and place the slip in a hat. What is the probability that at least 2 of the slips in the hat have the same number written on them?

Solution:
5. Solve the given linear programming problem using a corner point table:

Maximize and minimize $P = 10x_1 + 30x_2$

subject to

\begin{align*}
2x_1 + x_2 & \geq 16 \\
x_1 + x_2 & \geq 12 \\
x_2 + 2x_2 & \geq 14 \\
& \geq 0.
\end{align*}

Solution:
6. You are given the following linear programming problem:

Maximize \( P = 15x_1 + 10x_2 \)

subject to

\[
\begin{align*}
2x_1 + x_2 & \leq 10 \\
x_1 + 3x_2 & \leq 10 \\
x_1, x_2 & \geq 0.
\end{align*}
\]

(a) Using slack variables, write the initial system.

(b) Write the initial simplex tableau, circle the first pivot, and identify the entering and exiting variables.