Find all equilibrium solutions of the given differential equation and determine if they are sinks, sources, or neither:

\[ x' = x^4 - x^2. \]

**SOLUTION:** The equilibria are solutions to \( x^4 - x^2 = 0 \), i.e., \( x = -1, 0, 1 \). Let \( f(x) = x^4 - x^2 \). Then \( f(x) > 0 \) for \( x \in (-\infty, -1) \) and \( x \in (1, \infty) \), and \( f(x) < 0 \) for \( x \in (-1, 0) \) and \( x \in (0, 1) \). It follows that the phase line looks like this:

\[ \begin{array}{c}
-1 & \quad 0 & \quad 1 \\
\end{array} \]

Therefore, \(-1\) is a sink, \(1\) is a source, and \(0\) is neither. The first two statements also follow from the derivative test \((f'(-1) < 0 \text{ and } f'(1) > 0)\), but **not** the last one! Here’s why:

**Example 1.** Consider \( x' = x^3 \). Let \( f(x) = x^3 \). Then \( 0 \) is the only equilibrium, and since \( f(x) < 0 \) for \( x < 0 \) and \( f(x) > 0 \) for \( x > 0 \), it is a source, even though \( f'(0) = 0 \).

**Example 2.** \( 0 \) is a sink for \( x' = -x^3 \), even though \( f'(0) = 0 \) (where \( f(x) = -x^3 \)).