Welcome to CAMCOS Reports Day Spring 2009
The Propagation of Light in Causal Set Theory

Spring 2009

Jake Askeland, Jonathan Baptist, Miranda Braselton, David von Gunten, Douglas Mathews, Duncan McElfresh, Cheuk Wong

In collaboration with NASA Ames Research Center, Dr. Jeff Scargle
CAMCOS Advisor: Slobodan Simić
Outline

1 Introduction
2 Space-time and Light
3 Causal Sets
4 Feynman
5 Our Discrete Model
6 Johnston
7 Goal and Results
8 Computation
9 The Future
Einstein discovered Relativity.
Relativity is a physical theory that explains how gravity works.
Relativity gives good results at large distances.
It describes how planets move and why solar systems are shaped the way they are.
Introduction: Quantum Mechanics

- Quantum Mechanics describes how tiny things interact.
- It describes how atoms move and are structured.
- Quantum Mechanics gives nondeterministic results, so predictions of events are in probabilities.
Introduction: Theories in Conflict

- Quantum Mechanics and Relativity don’t make sense together.
- They are inconsistent with each other, and yet, accurately describe real events in our universe.
- For the last 50 years, physicists have searched for a new theory that unifies these two.
Introduction: Gamma Ray Bursts
Introduction: Gamma Ray Bursts

- Gamma rays are made of particles of light, called photons, at their highest energies.
- Bursts happen about once per day, in places all over the universe.
- Gamma Ray Bursts are the brightest thing we know of, lighting up the entire universe!
- Scientists discovered gamma rays with higher energy took longer to arrive.
Introduction: FERMI Gamma Ray Space Telescope

- FERMI launched June 11, 2008 to investigate this phenomenon.
- FERMI collects data on photons that traveled billions of light years.
Is the speed of light really constant?

- The data from FERMI may suggest the speed of a photon is dependent on its energy.
- Causal Set Theory (Sorkin, et al.) is a physical theory that may provide an explanation for this.
- Causal Set Theory is a theory intending to unify Quantum Mechanics and Relativity.
- As such, we can only compute the probabilities of events.
Introduction: Our Model

- We use a mathematical framework based on Causal Set Theory.
- Our goal was to determine if the speed of light is constant.
- Comparing the model with the FERMI data could result in new understanding.
- If the model fits, great! If it doesn’t, great?
- Let’s take a closer look at what we have done this semester...
Space-time

Definition

Space-Time is a mathematical model of the universe combining space and time.

- Any event that occurs in the universe has a specific location in space-time
- The space-time we observe has 3 spatial dimensions and one temporal dimension
- Thus Space-Time has 4 dimensions, or 3+1 D
1+1D Space-time
Proper Time

- Time passes differently for particles with different velocities.
- Proper time is $\tau = \sqrt{t^2 - x^2}$.
- $\tau$ is proper time, time as perceived by the blue particle.
- $t$ is time perceived by the red particle.
- $x$ is distance away from the red particle.
Nature of Light

Definition

Light is electromagnetic radiation of any wavelength

- The light spectrum is made up of the different wavelengths of light from radio to gamma
- The smallest packet of light is called a photon
Light Spectrum

Increasing energy ➞
When a photon travels through a medium, such as air, water or glass, the speed of light is slowed.

Einstein postulated that photons travel at a constant speed, $c \approx 3 \cdot 10^8 \text{m/s}$ in vacuum.

One of the goals of this project is to explore the possibility that light in a vacuum does not, in fact, travel at a constant rate, but actually varies in speed depending on the energy of the photon.
Light Cone

Definition

The light cone \((L)\) of a point \(P\) is the set of all points to and from which a particle can reach or be reached while traveling at the speed of light or less.
Shows where a particle may travel to in space with respect to time
- The boundaries of the cones show the path of a photon traveling at the speed of light.
- The slope of these boundaries are ± the speed of light.
Theory of Causal Sets

Initiated by Sorkin (1987):
- Attempts to unify Relativity and Quantum Mechanics
- An alternative to string theory
- Space-time is discrete
- Gives up the properties of a continuous universe
Definition of a Causal Set

**Definition**

A causal set, $S$, is a discrete set equipped with a relation of partial order.

- The discrete set is spacetime.
- The partial order describes whether a point $P$ can influence point $Q$ in spacetime.
Discrete Space-time Analogy

Film analogy:
- Like the universe, a film is perceived to be continuous
- Each second of film consists of multiple frames
- Fast sequencing produces the illusion of continuous motion
- Similarly, discrete space-time is perceived as continuous
Space-time Is Discrete

- No distance, area, volume, etc. in the usual sense.
- In discrete space-time, these are determined by the combinatorics of the set.

\[ A \neq L \times W \]
Graphical Representation of a Causal Set

Easy to represent as graphs of points and arrows between them:

- The vertices are points in space-time
- The edges, or arrows, are transitions between the points
Causal relations and events obey intuitive laws of physics:

- No going backwards in time
- No self-causing events (no loops)
Light Cone With Causal Relations

The Future
Goal and
Our Discrete
Causal Sets

Introduction
Space-time
and Light
Causal Sets
Feynman
Our Discrete
Model
Johnston
Goal and
Results
Computation
The Future

Light Cone With Causal Relations

Can be influenced by event P

Not causally connected to P

Can influence event P
Feynman Path Sum

Feynman plus path = Feynman Path Sum
Quantum Mechanics and Feynman Path Integral

- Quantum Mechanics works differently from Classical Mechanics.
- Feynman Path Integral is one of the main approach to Quantum Mechanics.
- In our model, we applied something similar to Feynman Path Integral (Feynman Path Sum) to Causal Set Theory.
In Quantum Mechanics, one of the main ideas regarding particles is probability amplitude.

We can find the probability of a particle traveling from point A to B by taking the absolute square of a complex number.

This complex number is called the probability amplitude, denoted by $\Psi$.

\[\text{prob}(A \leadsto B) = |\Psi|^2\]
Feynman Path Sum

Feynman (1948) developed Feynman Path Integral as an approach to Quantum Mechanics. Since Causal Set Theory deals with discrete space time, we will be using the discrete version of Feynman Path Integral, Feynman Path Sum instead.

**Definition**

Path Sum is the summation of the probability amplitudes of all possible paths for a particle to travel from point $A$ to point $B$.

$$\text{prob}(A \rightsquigarrow B) = \left| \sum_{\gamma} \Psi(\gamma) \right|^2$$

where $\gamma$ runs over all possible paths from $A$ to $B$. 
Feynman Path Sum, cont.

- The probability amplitudes for all paths that go from point A to point B are added together.
- The probability of a particle moving from point A to point B is the absolute square of the path sum:

\[
prob(A \rightarrow B) = \left| \sum_{\gamma} \Psi(\gamma) \right|^2
\]

where \( \gamma \) runs over all possible paths from A to B
The Propagation of Light in Causal Set Theory

Introduction
Space-time and Light
Causal Sets
Feynman
Feynman Path Sum
All Paths
Our Discrete Model
Johnston
Goal and Results
Computation
The Future

**Feynman Path Sum, cont.**

\[
\text{prob}(A \leadsto B) = \left| \sum_{\gamma} \Psi(\gamma) \right|^2
\]

where \(\gamma\) runs over all possible paths from A to B.
What Are All Paths?

- Following Feynman’s approach:
  - All paths include paths that “go backwards” in time
  - All paths include paths that move outside the light cone.
To visualize the causal set mathematically we create an adjacency matrix using the causal connections.

\[
\text{Adj} = \begin{pmatrix}
0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]
The amplitude matrix contains the probability amplitudes for single jumps.

The entries are defined by $\Psi = e^{iS}$ where

$$S = S[\text{Energy, Starting Point, Ending Point}]$$

$S$ is called the action and is pretty much proper time.
Amplitude Matrix Example

The entry in row 1, column 2 contains the probability amplitude, $e^{iS_1}$, for a single jump from point 1 to point 2.

$$A = \begin{pmatrix}
0 & e^{iS_1} & e^{iS_2} & e^{iS_3} \\
0 & 0 & 0 & e^{iS_4} \\
0 & 0 & 0 & e^{iS_5} \\
0 & 0 & 0 & 0
\end{pmatrix}$$
The probability amplitude of a photon moving from point 1 to 4 using all paths is
\[ \Psi = e^{i(S_3)} + e^{i(S_1 + S_4)} + e^{i(S_2 + S_5)} \cdot \gamma(1,4) \cdot \gamma(1,2) \cdot \gamma(2,4) \cdot \gamma(1,3) \cdot \gamma(3,4) . \]

\[ A = \begin{pmatrix}
0 & e^{iS_1} & e^{iS_2} & e^{iS_3} \\
0 & 0 & 0 & e^{iS_4} \\
0 & 0 & 0 & e^{iS_5} \\
0 & 0 & 0 & 0
\end{pmatrix} \]
Path Sums Using Matrices

- Each \( n^{th} \) power of \( A \) gives the probability amplitude of a path of length \( n \)
- We take the infinite sum of the amplitude matrix to create the propagator matrix \( K \)

\[
K = A + A^2 + \ldots = (I - A)^{-1} - I
\]
Applying this formula to our example gives the following:

\[ K = A + A^2 + \ldots = \begin{pmatrix} 0 & e^{iS_1} & e^{iS_2} & \Psi \\ 0 & 0 & 0 & e^{iS_4} \\ 0 & 0 & 0 & e^{iS_5} \\ 0 & 0 & 0 & 0 \end{pmatrix} \]

Where \( \Psi = e^{iS_3} + e^{i(S_1+S_4)} + e^{i(S_2+S_5)}. \)

- In practice our matrices are 800 by 800 so \( \Psi \) becomes crazy
The Propagation of Light in Causal Set Theory

Introduction

Space-time and Light

Causal Sets

Feynman

Our Discrete Model

Adjacency Marticies

Amplitude Matrix

Propagator Matrix

Johnston

Goal and Results

Computation

The Future

Intermission
Summary

What you learned so far:
- Physics background
- Causal Set Theory
- Feynman Path Sum

What‘s to come:
- Previous work
- Our project goals
- Results
- Computational implementation of our model
- Further research
The work of Johnston (Imperial College) has some similarities with ours. However:

- His method is different (e.g., he doesn’t allow all paths in his calculation).
- His goal is different (to solve the Klein-Gordon equation).

We implemented Johnston’s model, but we found that his approach to Feynman Path Sum and Causal Set does not seem to be suited to examining the question of whether the speed of light is constant.
In spring 2008, CAMCOS built a simulated environment for experiments.

- Discrete spacetime
- Feynman path sums
- Matrix models

This is what we do:

1. Define how our photons should move - different definitions of $S$ - the action.
2. Calculate the speed of our photons, by observing where they land on the detector.
Using this framework, many different definitions of $S$ were used.

- Unfortunately, results were inconclusive.
- Experiments showed that photons traveled at many speeds, but did not "prefer" standard $c$. 
What We Want

We want our experimental results to agree with FERMI’s observations.

- Light usually travels at $2.99 \times 10^8$ meters per second.
- But a photon’s speed can vary slightly, depending on its energy.
What We Want

In this experiment, 25% of all photons arrived at the detector with speed $c$.

The peak indicates that most photons still travel at standard $c$. 
Example Results

**Figure:** Probability vs. Velocity for Energy Scaling A

- **Velocity (rel. to c) vs. Probability and 1.0 * Energy**

- **Peak!**
Example Results

Figure: Probability vs. Velocity for Energy Scaling B

Velocity (rel. to c) vs. Probability and 1.0 - 1x10^-7 * Energy

Peak!
Example Results

**Figure:** Probability vs. Velocity for Energy Scaling C

- Velocity (rel. to c) vs. Probability and $1.0 - 1 \times 10^{-6} \times$ Energy

Peak!
Example Results

What you just saw:

- There is a distinct peak.
- We have not yet observed that this peak changes with a photon’s energy.
- When we ran our experiment with Johnston’s approach, but did not see a conclusive peak.
Project accomplishments

- Tried different several different formulas for $S$.
- Improved our experiment runtime by a factor of 10.
- Began using a histogram to represent our probabilities.
- Observed consistent results with many different configurations.
- Changed the code to allow for 2 and 3 space dimensions.
Why is it hard to understand and describe these probabilities?

- Our universe is really big,
- There are a lot of points in space-time a photon can be.
- So “All paths” means a lot of calculations.
- Imagine \( \psi = e^{i(S_3)} + e^{i(S_1+S_4)} + e^{i(S_2+S_5)} + \ldots \) for more points than you can imagine!
In a single run of our model universe:

- We restrict ourselves to 800 space-time points.
- That’s still 640,000 matrix entries in each of 4 matrices.
- We then compute tens of millions of paths!
- But at least our $\psi$ is manageable...
ψ =
\[ e^{i(S_1)} + e^{i(S_1+S_4)} + e^{i(S_2+S_5)} + e^{i(S_8+S_3)} + e^{i(S_8+S_1)} + e^{i(S_8+S_3)} +
\]
\[ e^{i(S_2)} + e^{i(S_4+S_7)} + e^{i(S_7+S_8)} + e^{i(S_9+S_2)} + e^{i(S_4+S_3)} + e^{i(S_4+S_2)} +
\]
\[ e^{i(S_3)} + e^{i(S_2+S_1)} + e^{i(S_2+S_4)} + e^{i(S_9+S_4)} + e^{i(S_6+S_1)} + e^{i(S_7+S_4)} +
\]
\[ e^{i(S_4)} + e^{i(S_1+S_6)} + e^{i(S_4+S_5)} + e^{i(S_4+S_1)} + e^{i(S_2+S_5)} + e^{i(S_6+S_4)} +
\]
\[ e^{i(S_5)} + e^{i(S_4+S_8)} + e^{i(S_3+S_1)} + e^{i(S_8+S_8)} + e^{i(S_1+S_2)} + e^{i(S_4+S_7)} +
\]
\[ e^{i(S_6)} + e^{i(S_8+S_2)} + e^{i(S_5+S_7)} + e^{i(S_2+S_7)} + e^{i(S_0+S_7)} + e^{i(S_8+S_9)} +
\]
\[ e^{i(S_7)} + e^{i(S_7+S_0)} + e^{i(S_6+S_6)} + e^{i(S_3+S_3)} + e^{i(S_6+S_6)} + e^{i(S_9+S_3)} +
\]
\[ e^{i(S_8)} + e^{i(S_9+S_1)} + e^{i(S_8+S_3)} + e^{i(S_4+S_6)} + e^{i(S_8+S_5)} + e^{i(S_3+S_2)} +
\]
\[ e^{i(S_9)} + e^{i(S_4+S_6)} + e^{i(S_3+S_7)} + e^{i(S_5+S_3)} + e^{i(S_4+S_4)} + e^{i(S_7+S_5)} +
\]
\[ e^{i(S_1)} + e^{i(S_5+S_7)} + e^{i(S_1+S_8)} + e^{i(S_7+S_4)} + e^{i(S_3+S_3)} + e^{i(S_5+S_7)} +
\]
\[ e^{i(S_3)} + e^{i(S_6+S_9)} + e^{i(S_0+S_9)} + e^{i(S_9+S_2)} + e^{i(S_7+S_8)} + e^{i(S_6+S_8)} +
\]
\[ e^{i(S_5)} + e^{i(S_2+S_4)} + e^{i(S_9+S_3)} + e^{i(S_7+S_7)} + e^{i(S_4+S_0)} + e^{i(S_2+S_9)} +
\]
\[ e^{i(S_7)} + e^{i(S_1+S_3)} + e^{i(S_5+S_2)} + e^{i(S_1+S_9)} + e^{i(S_2+S_3)} + e^{i(S_1+S_7)} +
\]
\[ e^{i(S_9)} + e^{i(S_0+S_6)} + e^{i(S_6+S_6)} + e^{i(S_4+S_3)} + e^{i(S_5+S_2)} + e^{i(S_7+S_6)} +
\]
\[ e^{i(S_2)} + e^{i(S_9+S_2)} + e^{i(S_3+S_0)} + e^{i(S_1+S_4)} + e^{i(S_6+S_6)} + e^{i(S_9+S_5)} +...\]
Comparison to previous models

Previous work on this idea has been done without experimental modeling:

- The run-time of naive models would be unbounded, they could continue for lifetimes, or just hours.
- Thus, there would be no bounds on the size or quantity of output, so data would be unwieldy.
- And so, of course, one would have no idea whether light in the model tends to travel at what we call the speed of light.
Comparison to CAMCOS 2008

At 800 points in a single run of the experimental model:

- The run completes in 38.1 seconds.
- Files outputted during the run total 14.7 megabytes.
- A distribution of velocities and their probabilities is unrecognizable or unreproducible.
- If we amassed results from this model it would take over 2 hours and nearly 3 gigabytes of hard drive space to get enough data for a reasonable histogram (200 runs).
At 800 points in a single run of our experimental model:

- A run completes in 4.12 seconds.
- We append only the data we need to a file of about 5 kilobytes.
- Our model can aggregate results,
- So we can see a consistent velocity/probability distribution which appears to be Gamma\(^*\)
- It takes 13.8 minutes and about 1 megabyte of hard drive space for the same 200-run histogram.

\(^*\)A Gamma distribution has a skew to the left based on a shape parameter.
### Computational Analysis

Comparison @ 800 pts:

<table>
<thead>
<tr>
<th>Year</th>
<th>Run time</th>
<th>Output size</th>
<th>Velocity distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 2008</td>
<td>N/A</td>
<td>N/A</td>
<td>None</td>
</tr>
<tr>
<td>2008</td>
<td>38.1 seconds</td>
<td>14.7 mbytes</td>
<td>Unrecognizable</td>
</tr>
<tr>
<td>2009</td>
<td>4.12 seconds</td>
<td>5 kbytes</td>
<td>Humanly recognizable</td>
</tr>
</tbody>
</table>
The code we inherited has been modified and now allows for:

- Tens of thousands of runs per day.
- An aggregation of resultant velocity/probability data.
- Distributed computing, thanks to independent runs.
- Histogram* display of velocity/probability data.

*Histograms can condense lots of data into a simple, viewable set of statistical properties.
Matrix visuals were used to debug the model. They give the order of magnitude of our data throughout the calculations.
Future directions

- The algorithms generating proper time and velocity should be changed to use the combinatorics of the causal set to be true to the Causal Set Theory approach.

- We conjecture that the speed of light approaches an apparent constant as the model is made larger, approximating the universe better. Testing this has not been done yet.
Some last minute experiments with our choice of $S$ in $e^{iS}$ may reveal more about the project. We would like to explore $S$ more.

FERMI has been collecting data that we now have a valid model to compare with. We would like to explore that comparison.

The linear algebraic properties of our matrices may reveal more underlying results about our experiment.

More experimentation and analysis is needed with the new 3+1D experiment set up.
Continuation of this project is expected to occur in either Fall 2009 or Spring 2010 semester.

Past CAMCOS projects include those in the areas of Statistics, Linear Algebra, Dynamical Systems, and Physics

CAMCOS is always looking for new, exciting and challenging projects

To learn more about CAMCOS, as a sponsor or participant, email Tim Hsu at hsu@math.sjsu.edu or visit our website at http://www.math.sjsu.edu/camcos/
Stay tuned for our soon-to-be posted paper containing all the technical details left out of this presentation.

The paper will be posted on our advisor’s website:

http://www.math.sjsu.edu/~simic
Thanks

Thank you for your attention.

Any questions?
The Propagation of Light in Causal Set Theory

Introduction
Space-time and Light
Causal Sets
Feynman
Our Discrete Model
Johnston
Goal and Results
Computation
The Future

Lunch at Flames

* Flames
San Fernando

King Library

SJSU Campus

P
San Salvador

4th St.