Welcome to CAMCOS Reports Day Spring 2011
Particle Levitation, Trajectories, and Mass Transfer in a Charged Binary Asteroid System

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Motivation

Model Overview
- Definition of Levitation Conditions
- Visualization of 4 Body Problem
- Definition of Forbidden Regions

Intermission

Code Optimization

Levitation Maps

Charged particle trajectories
- Types of trajectories
- Examples

Monte Carlo Simulations
- Overview of methods and implementation
- Presentation of results
Study the motion of loose particles in a charged binary asteroid system.
Motivation

- Characterization of asteroids
  - What is the size and shape of an asteroid?
  - What is the surface composition of an asteroid?
- Space mission planning
  - NEAR Shoemaker (1996-2001)
  - Lunar Atmosphere and Dust Environment Explorer (2013)
  - Orion Asteroid Mission
Accomplishments of Last Semester

- Derived equations of motion for 2 asteroids, particle, sun
- Added charge to the system via the sun
- Produced contour plots of the electrical potential
- Produced an animation of the changing forbidden regions
- Identified where/when particles lose contact with surface of the asteroid
- Plotted possible paths of loose particles
Goals for this Semester

- How can we optimize code for better runtime efficiency?
- Can we incorporate the analytic gradient into our model?
- How does analytic gradient affect levitation maps & trajectories?
- What is the average percentage of transfer, return and escape trajectories?
- How long does a particle take to move along a typical trajectory?
Charging Mechanism

- **Photoelectric effect**
  - Light from the sun causes the surface and the particle to get charged simultaneously
  - They become charged with the same type of charge (+)
  - This causes the particle to be repelled from the surface
Particle Levitation

- **Definition of Particle Levitation**
  - The interaction of the gravitational force and electrical forces on particles on an asteroid’s surface may cause a particle to “levitate” from the surface.
What Is Levitation?

- Asteroids have a low gravity environment
- Electric fields could be significant

Asteroid Itokawa-September 12, 2005
Levitation Conditions of free particles

- $F_n$: net outward force acting on loose particle
- General Levitation Condition: $F_n > 0$
- $F_n$ includes contributions from
  - Electrical force acting on the particle
  - Gravitational forces of other bodies in the system
  - Centripetal force due to spinning asteroid
  - Inertial forces due to rotation of frame, centripetal and Coriolis forces.
Model Description

- **Model:**
  - 4 Bodies: Sun, two asteroids, particle
  - asteroid mass $>>$ particle mass

- **System dynamics:**
  - Binary asteroid system orbits the sun ($\Omega$)
  - The two asteroids orbit their center of mass ($\omega$)
  - Each asteroid rotates about its own axis ($\Omega_s$)
Model Description

- Rotating reference frame

- Gravitational and electric forces from asteroids
  \[ ma = F_{g1} + F_{g2} + F_{e1} + F_{e2} \]

- Fictitious forces from noninertial reference frame
  - Centripetal and Coriolis forces due to rotation of frame
  - Centripetal force due to heliocentric orbit
  \[ a = \ddot{R} + \omega \times (\omega \times \rho) + 2\omega \times (\dot{\rho})_r + (\ddot{\rho})_r \]
Incorporating Electric Forces

- LaPlace’s Equation: $\Delta V = 0$
  - Boundary Conditions: 5V on sunlit hemisphere
    -1000V on dark hemisphere

- Electric field around asteroid: $E = -\nabla V$

- Electric force on charged particle: $F = qE$
Binary System Electric Field

Direction to Sun
Forbidden Regions

Instantaneous Forbidden Region

A region in space the particle cannot reach at a specified time, given its kinetic energy at that time.

- Jacobi Integral: \( J(t) = \frac{KE}{m} - U(t) \)
- Restriction: \( \frac{KE}{m} = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \geq 0 \)
- This divides the space into different regions:
  - Admissible region: \( J(t) + U(t) \geq 0 \)
  - Forbidden region: \( J(t) + U(t) < 0 \)
- Forbidden region varies with time
Where We Started This Semester

- **Addressing software problems**
  - Gradient calculation
  - Trajectory failures
  - Performance issues

- **Updating the model to include:**
  - 3D forbidden regions
  - 3D levitation maps
  - 3D trajectories
Software Problems

- We implemented the analytical gradient and it improved levitation maps and trajectories.
- Some trajectories still failed mysteriously in MATLAB’s ODE45 tool.
- Computation limited our work.
Mysterious Trajectory Failures in ODE45
Performance

We tackled performance problems:

- Vectorization - MATLAB term for making computations in parallel helped with the following:
  - Calculating forbidden regions in 3D space.
  - Particle levitation maps.
- Trajectories still slow!
Vectorization

What is vectorization?

- **Scalar or Serial Processing**
  - One person sorts a desk of cards
- **Vector or Parallel Processing**
  - Split the deck into four stacks, have four people sort it at the same time
3D Forbidden Regions
# Runtime Differences

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<th>Process</th>
<th>Scalar points</th>
<th>Scalar time</th>
<th>Performance</th>
</tr>
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<tr>
<td>Precalculate U</td>
<td>65K</td>
<td>95 s</td>
<td>690 points/s</td>
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<tr>
<td>Levitation</td>
<td>10K</td>
<td>3321 s</td>
<td>3.01 points/s</td>
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<tr>
<td>Forbidden region</td>
<td>56 million</td>
<td>2400 s</td>
<td>23K points/s</td>
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<table>
<thead>
<tr>
<th>Process</th>
<th>Vector points</th>
<th>Vector time</th>
<th>Performance</th>
<th>Factor</th>
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<tbody>
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<td>Levitation</td>
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<td>77 s</td>
<td>129.8 points/s</td>
<td>43x</td>
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<td>Forbidden region</td>
<td>4 billion</td>
<td>14400 s</td>
<td>280K points/s</td>
<td>12x</td>
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</table>
5 minute Intermission
Particle Levitation Map

- Particles will Levitate when $F_n > 0$ (the outward normal force is greater than zero)
- Levitation map drawn as a Mercator map of the asteroid’s surface, like a flat map of the Earth.
There is no general map of levitation regions.

F = qE, Electric force depends on the charge of the object.

Levitation therefore depends on charge-to-mass ratio of object.
Levitation Maps dependent on Charge-to-Mass Ratio

- Levitation occurs in green regions.

![Graphs showing levitation maps with different charge-to-mass ratios](image-url)
Our system changes as a function of time, therefore our levitation map changes too.
Sometimes things are lost when projecting onto 2D maps, so we want to look at 3D maps instead.
If we look up close we see that the strongest forces occur at the terminator regions.
Our potential function is an infinite sum of Legendre polynomials.

To find the gradient, we took a finite difference approximation last semester.

Last semester’s model, based on the numerical calculation of the gradient, gave some strange results.

We wanted to address this problem, but ran out of time.
One key goal for this semester was incorporating the analytic gradient into our model.

The model now uses the analytic formula to calculate the gradient.

This change has affected both levitation maps and trajectories.

Resulted in more intuitive levitation maps.

Far more confident in the accuracy of our model.
Trajectories

- Our model uses the ordinary differential equations solver ODE45 to calculate trajectories.
- This model solves our system, and gives us the path coming from a point in space, with starting velocity.
- We can now give accurate starting positions and velocities on the surface of our asteroids using the levitation maps and spin rate of the system.
- For a particle levitating, we have three possible outcomes: Return, Transfer or Escape.
A Sample Return Orbit

Return Trajectory: The particle comes back to the same asteroid it levitated from
A Sample Transfer Orbit

Transfer Trajectory: The particle impacts the other asteroid
Complex Trajectories

- While behavior can be separated cleanly into three cases
- Trajectories themselves can be quite complicated
- Here are some examples
The Forbidden Region and Trajectories

- We now have visual tools to look at the 3d forbidden region in time along with trajectories.
- As the particle travels along its path, the red forbidden region changes.
- Some cusps in the trajectory are because the particle approaches a forbidden region and must change direction quickly.
What are Monte Carlo Simulations?

- MC methods are techniques that use random sampling to examine a problem.
- Solve problems by observing the percentage of random input that obey some property
- Similar to rolling dice many times and seeing what we get
Reasons for Use

- Great for simulating phenomena
- Allows us to examine complicated problems
- Large systems can be sampled
General Algorithm

1. Define a domain of possible inputs
2. Generate inputs randomly from a probability distribution over the domain
3. Perform a deterministic computation on the inputs
4. Aggregate the results
MC Example: Area of a Circle

- Circle circumscribed by square
- Throw darts randomly at the figure
- Fraction of darts inside circle $\approx$ area of the circle
Algorithm for Our System

1. Define a domain of possible inputs
   (Surface of asteroid)
2. Generate inputs randomly from a probability distribution
   over the domain
   (Randomly populate the asteroid with virtual particles)
3. Perform a deterministic computation on the inputs
   (Numerical trajectory integration)
4. Aggregate the results
   (Determine return, transfer and escape statistics)
Algorithm for Our System

1. Generate random particles
2. Update position due to asteroid spin
3. Levitation check
   - Yes: Store Initial Conditions
   - No: Continue Tracking
4. Compute Trajectories and Classify
Dependence of Charge-to-Mass Ratio

Percentage of return trajectories for different charge-to-mass ratios.
Average transit time for a particle due to different charge-to-mass ratios.
Results of Monte Carlo Simulations

- Revolution time of asteroid system 16.5 hours
- 72000 random points generated & charge-to-mass ratio 1 C/kg

<table>
<thead>
<tr>
<th></th>
<th>Percentage</th>
<th>Avg. Transit Time (hrs)</th>
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<tbody>
<tr>
<td>Transfer</td>
<td>30.6%</td>
<td>10.02</td>
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<tr>
<td>Return</td>
<td>69.4%</td>
<td>4.85</td>
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<tr>
<td>Escape</td>
<td>&lt;.1%</td>
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Future Research

- Incorporate levitation and mass transfer into more general binary asteroid systems with irregularly shaped asteroids and non-circular orbits.
- Incorporate effect of solar winds on particle trajectories.