

## Calculating singular values using svds\_err

Leslie Foster  
Department of Mathematics  
San Jose State University  
4/18/2009

The routine `svds_err` is a minor modification of Matlab's `svds`. Matlab's `svds` calculates a few singular values and (optionally) singular vectors using Matlab's `eigs` applied to  $B = \begin{bmatrix} 0 & A \\ A' & 0 \end{bmatrix}$ . The routine `svds_err` also provides error bounds on the calculated singular values. This is done by using the bounds contained in, for example, Theorem 5.5, p. 205, Applied Numerical Linear Algebra, (SIAM Press, 1997) by James Demmel.

The location [http://www.math.sjsu.edu/singular/matrices/software/SJsingular/svds\\_err.m](http://www.math.sjsu.edu/singular/matrices/software/SJsingular/svds_err.m) contains the `svds_err` code.

### Use of `svds_err`:

`SVDS_ERR` Find a few singular values and vectors and provides error bounds.

If  $A$  is  $M$ -by- $N$ , `SVDS_ERR(A,...)` manipulates a few eigenvalues and vectors returned by `EIGS(B,...)`, where  $B = \begin{bmatrix} \text{SPARSE}(M,M) & A \\ A' & \text{SPARSE}(N,N) \end{bmatrix}$ , to find a few singular values and vectors of  $A$ . The positive eigenvalues of the symmetric matrix  $B$  are the same as the singular values of  $A$ .

`S = SVDS_ERR(A)` returns the 6 largest singular values of  $A$ .

`S = SVDS_ERR(A,K)` computes the  $K$  largest singular values of  $A$ .

`S = SVDS_ERR(A,K,SIGMA)` computes the  $K$  singular values closest to the scalar shift `SIGMA`. For example, `S = SVDS_ERR(A,K,0)` computes the  $K$  smallest singular values.

`S = SVDS_ERR(A,K,'L')` computes the  $K$  largest singular values (the default).

`S = SVDS_ERR(A,K,SIGMA,OPTIONS)` sets some parameters (see `EIGS`):

Field name	Parameter	Default
<code>OPTIONS.tol</code>	Convergence tolerance: $\text{NORM}(A*V-U*S, 1) \leq \text{tol} * \text{NORM}(A, 1)$ .	1e-10
<code>OPTIONS.maxit</code>	Maximum number of iterations.	300
<code>OPTIONS.disp</code>	Number of values displayed each iteration.	0

`[U,S,V] = SVDS_ERR(A,...)` computes the singular vectors as well.

If  $A$  is  $M$ -by- $N$  and  $K$  singular values are computed, then  $U$  is  $M$ -by- $K$  with orthonormal columns,  $S$  is  $K$ -by- $K$  diagonal, and  $V$  is  $N$ -by- $K$  with orthonormal columns.

`[U,S,V,FLAG] = SVDS_ERR(A,...)` also returns a convergence flag.

If EIGS converged then  $\text{NORM}(A*V-U*S,1) \leq \text{TOL} * \text{NORM}(A,1)$  and FLAG is 0. If EIGS did not converge, then FLAG is 1.

$[U,S,V,FLAG,ERR\_SVALS] = \text{SVDS\_ERR}(A,...)$  also returns ERR\_SVALS, a vector with the same length as the number diagonal entries in S.

$$|S(i,i) - (\text{a singular value of } A)| \leq \text{ERR\_SVALS}(i). \quad (1)$$

It is likely, but not guaranteed, that the singular value in this equation is the  $i$ th singular value of A.

Note: SVDS\_ERR is best used to find a few singular values of a large, sparse matrix. To find all the singular values of such a matrix, SVD(FULL(A)) will usually perform better than SVDS\_ERR(A,MIN(SIZE(A))).

Example:

```
load west0479
sf = svd(full(west0479));
[U,S,V,flag,err_s1] = svds_err(west0479,10); s1 = diag(S);
[U,S,V,flag,err_s0] = svds_err(west0479,10,0); s0 = diag(S);
[U,S,V,flag,err_s2] = svds_err(west0479,10,2); s2 = diag(S);
```

s1 will be a vector of the 10 largest singular values,  
err\_s1(i) will provide error bounds for s1(i) in the sense of equation (1),  
s0 will be a vector of the 10 smallest singular values,  
err\_s0(i) will provide error bounds for s0(i) in the sense of equation (1),  
s2 will contain the 10 singular values of west0479 which are closest to 2 and,  
err\_s2(i) will provide error bounds for s2(i) in the sense of equation (1),

See also SVD, EIGS.

## References:

Zhaojun Bai, James Demmel, Jack Dongarra, Axel Ruhe and Henk van der Horst, Templates for the Solution of Algebraic Eigenvalue Problems, SIAM, Philadelphia, 2000.

James Demmel, Applied Numerical Linear Algebra, SIAM, Philadelphia, 1997.

R. B. Lehoucq, Danny C. Sorensen and C. Yang. ARPACK Users' Guide: Solution of Large-Scale Eigenvalue Problems with Implicitly Restarted Arnoldi Methods. SIAM, Philadelphia, 1998.

Beresford. N. Parlett, The Symmetric Eigenvalue Problem, Prentice-Hall, Englewood Cliffs, NJ, 1980. Reprinted as Classics in Applied Mathematics 20, SIAM, Philadelphia, 1997.

Hong Zhang, Barry Smith, Michael Sternberg and Peter Zapol, SIPs: Shift-and-Invert Parallel Spectral Transformations, ACM Transactions on Mathematical Software, 33:1-19, 2007.