PLIMPTON 322

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Plimpton 322 records information on 15 right-angled triangles. The exact method used for the table is not known.

It is known how to parameterize right triangle sides in terms of integer parameters a, b: the sides are $x = a^2 - b^2$, $y = 2ab$ and the hypotenuse is $z = a^2+b^2$ satisfying the Pythagorean Theorem $x^2+y^2 = z^2$.

In Babylonian times one used base 60. Consider the numbers $w = \frac{a}{b}$ having prime factors in $\{2, 3, 5\}$. For our description let $w = 2^i 3^j 5^k$; the numerator of this fraction we call a and the denominator we call b. If we restrict $-5 \leq i \leq 6$, $-3 \leq j \leq 4$, $-2 \leq k \leq 3$ and $9/5 \leq w \leq 12/5$ then we get exactly the 15 Plimpton solutions.

	x	z	i	j	k	w
1	1, 59	2, 49	2	1	-1	12/5
2	56, 7	1, 20 25	6	-3	0	64/27
3	1,16,41	1,50,49	-5	1	2	75/32
4	3,31,49	5,9,1	-1	-3	3	125/54
5	1,5	1,37	-2	2	0	9/4
6	5,19	8,1	2	-2	1	20/9
7	38, 11	59, 1	1	3	-2	54/25
8	13, 19	20,49	5	-1	-1	32/15
9	8,1	12, 49	-2	-1	2	25/12
10	1, 22, 41	2, 16, 1	-3	4	-1	81/40
11	45	1, 15				
11*	3	5	1	0	0	2
12	27, 59	48, 49	4	1	-2	48/25
13	2, 41	4, 49	-3	1	1	15/8
14	29, 31	53, 49	1	-3	2	50/27
15	28	53				
15*	56	1, 46	0	2	-1	9/5

Alternatively we may describe the Plimpton solutions as those with i, j, k as specified above and the angle $30^0 \leq \theta \leq 45^0$. Here, we use the angle θ, the arctangent of $\frac{x}{y}$. In this way we obtain 17 solutions: 15 fit nicely into the description for Plimpton 322; two of the solutions (numbers 11, 15) are equivalent fractions (using a factor of 15 or 2, respectively) to those used in Robson's version of Plimpton. The remaining two extra solutions are: $z = 5, 46, 49 = 20809$, $x = 2, 54, 1 = 10441$ corresponding to $w = \frac{5^3}{2^3 3^2}$. and $z = 5, 37 = 337$, $x = 2, 55 = 175$ corresponding to $w = \frac{2^4}{3^2}$.

References

[1] R. C. Buck, *Sherlock Holmes in Babylon*, Amer. Math. Monthly, 87 (1980), 335-345.

[2] Eleanor Robson, *Words and Pictures: New Light on Plimpton 322*, Amer. Math. Monthly, 109 (2002) 105-120

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