

A Totally Real Folding of the Regular Heptagon

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There are several origami constructions for the heptagon; see [4] for one example. However, the construction is a totally real construction [3], so we can achieve it with bisections and trisections of angles and constructions of perpendicular through a point not on a line. We modify the heptagon construction in [2]. The explanation of the trisection which we use below is discussed there.

Take a square piece of paper say 6 units on a side with center V and OV of length 1 on the horizontal midline OA_7 of the square.

Construct distance of $BV = 3\sqrt{3}$ on the vertical midline of the square. First construct A with AV of length 5 parallel to the vertical midline; bisect the angle of AV and midline; reflect A across this bisector to A' on the midline; $A'V$ has length $\sqrt{26}$; move A' to A_1 and reflect back to B so that the hypotenuse BO of $\triangle BOV$ has length $\sqrt{28}$.

Trisect $\angle BOV$ to get the point T as in Abe's trisection (OT is the trisection with the base OB).

Reflect B to B_2 across OT . Reflect B_2 across OV to get B_3 .

We fold the heptagon as follows.

- (1) Fold A_7 across VB_3 to get G_7 .
- (2) Reflect G_7 across OA_7 to get B_7 .
- (3) Construct 180 degree symmetry of G_7 about V to G'_7
- (4) Bisect $\angle G'_7VB_3$, reflect G'_7 across this bisector to D_7
- (5) Reflect D_7 across VO to get vertex E_7 .
- (6) Reflect E_7 across the bisector of G'_7VB_3 (through B_7) to get F_7
- (7) Reflect F_7 across VO to get C_7 .

References

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- [3] Roger C. Alperin, *Trisections and Totally Real Origami*, Amer. Math. Monthly, vol. 112, no. 3, 200-211, March 2005
- [4] Benedetto Scimemi, *Regular Heptagon by Folding*, Proceedings of Origami, Science and Technology, ed. H. Huzita., Ferrara, Italy, 1990.

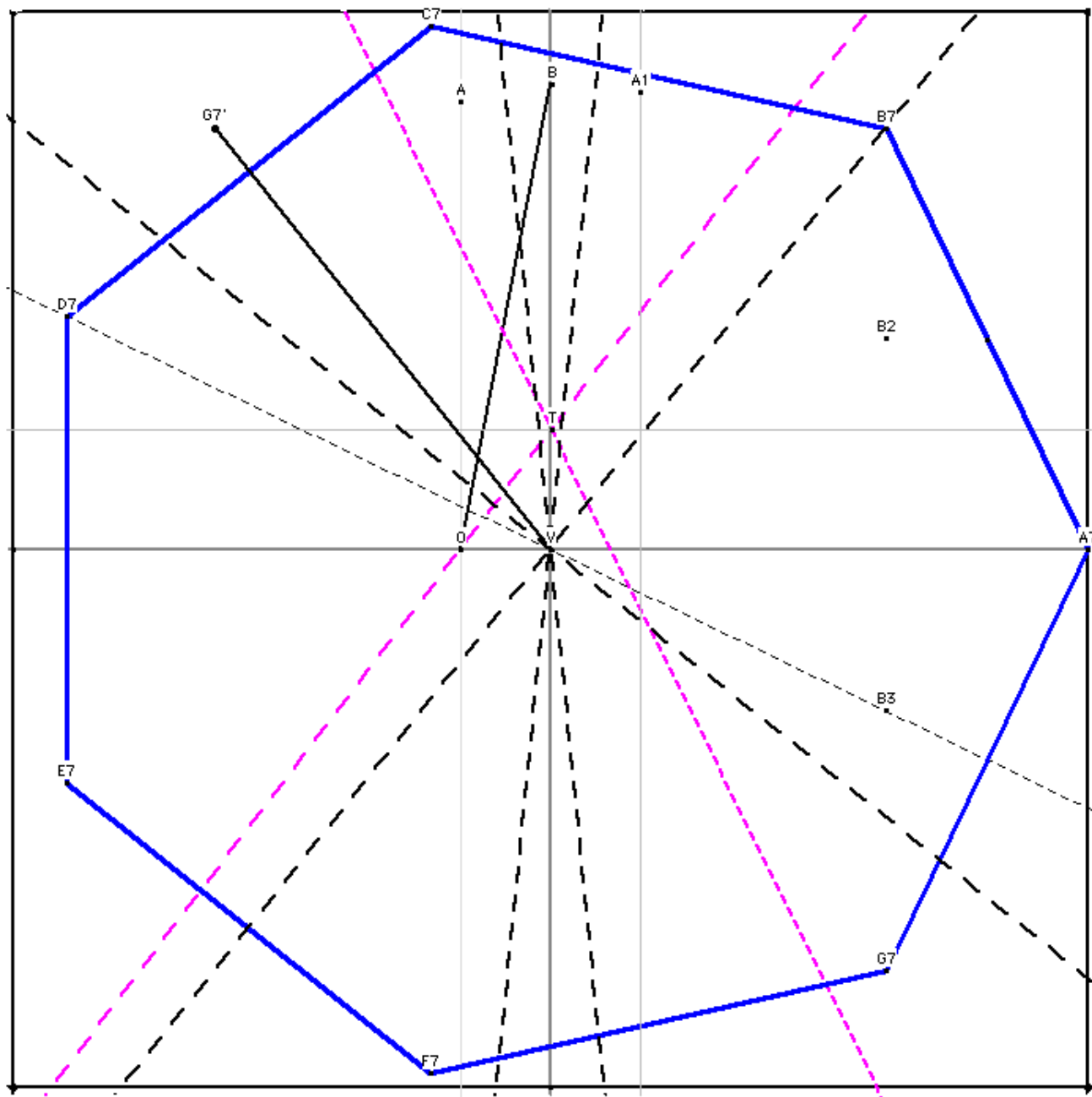


FIGURE 1. Folding the Regular Heptagon

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