

Computer Project: The Adjacency Matrix of a Graph

Name _____

Purpose: To learn about graph and adjacency matrix, to see how the powers of the adjacency matrix provide information about the graph and vice versa, and to apply these ideas.

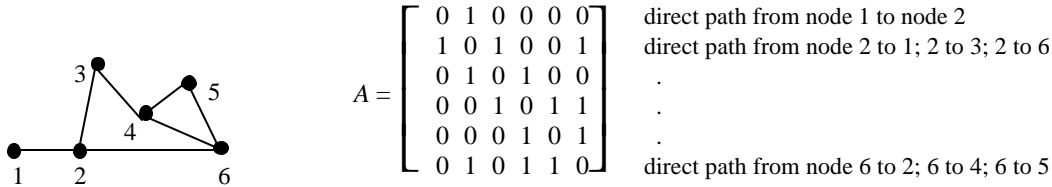
Prerequisite: Section 2.1

MATLAB functions used: +, ^; and **adjdat** from Lay's Toolbox

Definitions. A *graph* is a finite set of objects called nodes, together with some paths between some of the nodes, as illustrated below. A *path of length one* is a path that directly connects one node to another. A *path of length k* is a path made up of k consecutive paths of length one. The same length one path can appear more than once in a longer path; for example, 1--2--1 is a path of length two from node 1 to itself in the example below.

When the nodes have been numbered from 1 to n, the *adjacency matrix A* of the graph is defined by letting $a_{ij} = 1$ if there is a path of length one between vertices i and j and $a_{ij} = 0$ otherwise.

Example. Verify that the graph below has the matrix A shown as its adjacency matrix.



$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

direct path from node 1 to node 2
 direct path from node 2 to 1; 2 to 3; 2 to 6
 .
 .
 .
 direct path from node 6 to 2; 6 to 4; 6 to 5

Theorem (Interpretation of the powers of an adjacency matrix). If A is the adjacency matrix of a graph, then the (i,j) entry of A^k is a nonnegative integer which is the number of paths of length k from node i to node j in the graph.

1. (hand) To understand what the theorem says for the example above, let's carefully examine the (6,3) entry of A^2 . Using the "Row-Column Rule," the (6,3) entry of A^2 looks like $a_{61}a_{13} + a_{62}a_{23} + a_{63}a_{33} + a_{64}a_{43} + a_{65}a_{53} + a_{66}a_{63}$.

Evaluate each term in this expression (you finish): $(0)(0) + (1)(1) + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$.

Explain what each term in the sum above tells about paths of length 2 from node 6 to node 3. (For example, $a_{62}a_{23} = (1)(1) = 1$; this says that the length one paths 6---2 and 2---3 appear in the graph, and together they give one path from node 6 to node 3, of length 2.)

2. Type **adjdat** to get the matrix A above, then type **A^2, A^3** and record results:

$A^2 =$ $A^3 =$

(b) (hand) Notice that the (1,2) entry of A^2 is zero, so there are no paths of length two from node 1 to node 2. Verify this by studying the graph. Similarly, notice that the (6,6) entry of A^3 is two, so there are two paths of length three from node 6 to itself; study the graph to see that they are 6--4--5--6 and 6--5--4--6.

In the same way, study the matrices and the graph and answer:

What are the paths of length two from node 2 to itself? _____

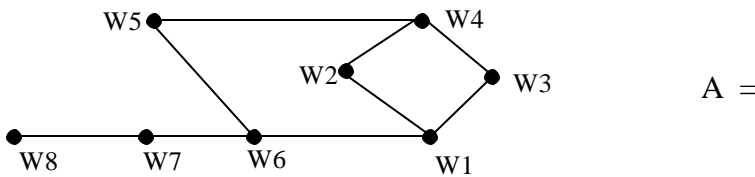
What are the paths of length three from node 3 to node 4? _____

Definition. When we have a graph, we will say that there is a *contact level k* between node i and node j if there is a path of length less than or equal to k from node i to node j.

3. (hand) Suppose A is the adjacency matrix of a graph. Explain why you must calculate the sum $A + A^2 + \dots + A^k$ in order to decide which nodes have contact level k with each other:

4. Eight workers, denoted $W1, \dots, W8$, handle a potentially dangerous substance. Safety precautions are taken but accidents do happen occasionally. It is known that if a worker becomes contaminated, s/he could spread this through contact with another worker. The following graph shows the level one contacts between the workers.

(a) (hand) Write the adjacency matrix A for the following graph:



(b) Store A , type $A + A^2 + A^3$ and record result: $A + A^2 + A^3 =$

Use this to answer the following questions. Which workers have contact level 3 with $W3$? _____

Which workers have contact level 3 with $W7$? _____

(c) Define what you mean by a worker being *dangerous*. Be very specific so anyone could decide whether a worker is "dangerous" according to your definition:

(d) Which workers are the most dangerous if contaminated? _____ Which are least dangerous?

Use your definition, and explain your answers. This part is important, but whatever you say is ok as long as it agrees with your definition. Use the back or attach an extra sheet.