1. Consider the differential equation \( u''(x) + u'(x) = 5 \) with \( u(0) = u(4) = 0 \). Divide the interval \( 0 \leq x \leq 4 \) into \( n = 4 \) subintervals. By approximating \( u''(x) \) and \( u'(x) \) find a system of 3 equations in 3 unknowns that will approximate the solution to the above differential equation. To approximate \( u'(x) \) use \( u'(x) = [-u(x-h) + u(x+h)] / (2h) \). Write your answer in matrix form \( Au = b \).

\[
\begin{align*}
\Delta x &= \frac{4-0}{4} = 1 \\
\therefore \quad u_i' &= \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} \\
\text{so} \quad \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} + \frac{-u_{i+1} + u_{i+1}}{\Delta x} &= 5 \\
\therefore \quad h &= 1 \\
\Rightarrow \quad 2u_{i+1} - 4u_i + 2u_{i-1} &= 10 \\
\Rightarrow \quad u_{i+1} - 4u_i + 3u_{i-1} &= 0 \\
\Rightarrow \quad u_0 = u_4 = 0 \\
\Rightarrow \quad i = 1: u_1 - 4u_1 + 3u_0 &= 10 \\
\Rightarrow \quad i = 2: u_2 - 4u_2 + 3u_1 &= 10 \\
\Rightarrow \quad i = 3: u_3 - 4u_3 + 3u_2 &= 10 \\
\Rightarrow \quad \begin{pmatrix} -4 & 3 & 0 \\ 1 & -4 & 3 \\ 0 & 1 & -4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \\ 10 \end{pmatrix}
\end{align*}
\]

3. (a) Find the one norm and infinity norm of \( A = \begin{pmatrix} 1 & -4 & 7 \\ 5 & -2 & 3 \\ 1 & 2 & -4 \end{pmatrix} \).

\[ \|A\|_1 = \text{col. sum norm} = \max(1, 5, 1, 4, 2, 4) = 5 \]
\[ \|A\|_\infty = \text{row sum norm} = \max(14, 7, 8, 5, 3, 12, 4) = 14 \]

(b) Discuss why, for large matrices, the one or infinity norm is preferable to the two matrix norm.

The two norm is difficult to calculate. It is

\[ \|A\|_2 = \text{largest eigenvalue} (A^TA)^{1/2}, \text{ much harder than } \|A\|_1, \|A\|_\infty \]

1. Suppose the singular value decomposition of the matrix \( A \) is

\[ A = UDV^T \]

\[ \begin{pmatrix} .5816 & -.5731 & -.5774 \\ .2055 & .7902 & -.5774 \\ .7871 & .2172 & .5774 \end{pmatrix} \begin{pmatrix} 8.2328 & 0 & 0 \ 0 & .4704 & 0 \ 0 & 0 & .977 \times 10^{-15} \end{pmatrix} \approx \begin{pmatrix} .619 & -.128 & .559 & -.536 \\ .166 & -.756 & .213 & .595 \\ .453 & .628 & .213 & .595 \\ .619 & 0.128 & -.772 & -.059 \end{pmatrix} \]

For the right hand side \( b = (7 \ 2 \ 9)^T \) one solution to \( Ax = b \) is \( x = (1 \ 1 \ 0 \ 1)^T \) (here \( ^T \) indicates transpose). Describe the set of all solutions, assuming the computations are done in approximately 15 or 16 digit arithmetic.

\[ x = (1) + (.559 \ .536 \ .536 \ .536)^T (\bar{x}) \]