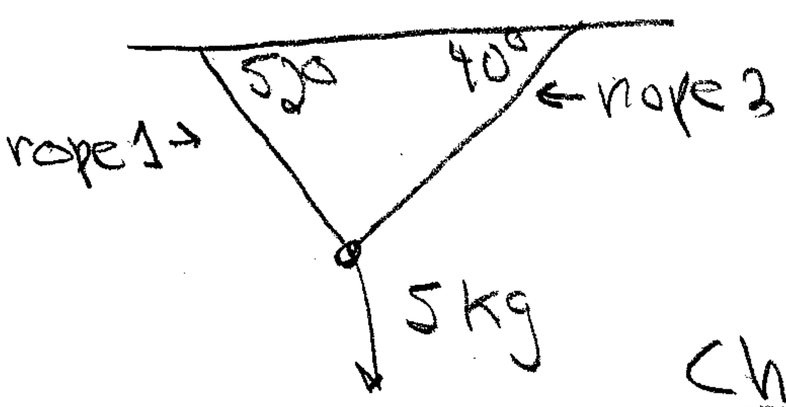
Ex Vectors Applied to Statics



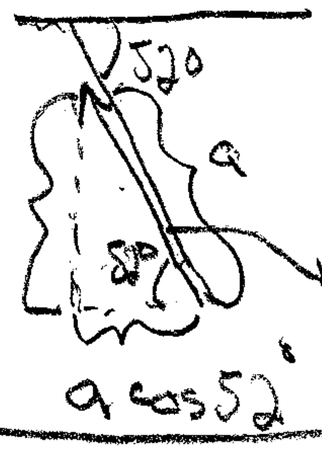
Find the tension along each rope when a 5kg mass is suspended as pictured.

Choose variables:

$\underline{T}_1$ = tension vector along rope 1, $a = |\underline{T}_1| =$ length $\underline{T}_1$

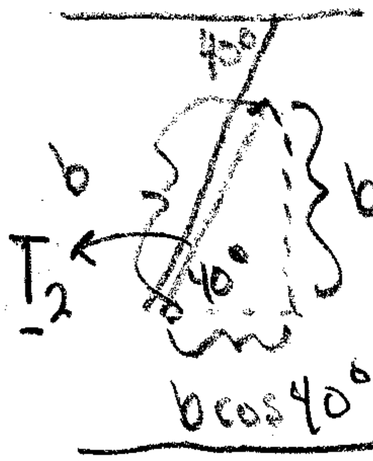
$\underline{T}_2$ = " " " 2, $b = |\underline{T}_2| =$ length $\underline{T}_2$

Express $\underline{T}_1$ in terms of a:



so $\underline{T}_1 = -a\cos 52^\circ \underline{i} + a\sin 52^\circ \underline{j}$

Express $\underline{T}_2$ in terms of b:



$\underline{T}_2 = b\cos 40^\circ \underline{i} + b\sin 40^\circ \underline{j}$

$\underline{F}_g$ = Force of gravity $= -mg\underline{j}$ $= -5\text{kg} \cdot 9.8 \frac{m}{sec^2}\underline{j}$

$= -49\,\underline{j}$ Newtons

$= -49\,\underline{0}$ (omitting units)

Use the law of statics: the net force is zero

so $\underline{0} = \underline{T}_1 + \underline{T}_2 + \underline{F}_g$ (eqn)

$\underline{0} = -a\cos 52^\circ \underline{i} + a\sin 52^\circ \underline{j} + b\cos 40^\circ \underline{i} + b\sin 40^\circ \underline{j} - 49\underline{j}$

$= (-a\cos 52^\circ + b\cos 40^\circ)\underline{i} + (a\sin 52^\circ + b\sin 40^\circ - 49)\underline{j}$

Therefore eqn 1: $-a\cos 52^\circ + b\cos 40^\circ = 0$

eqn 2: $a\sin 52^\circ + b\sin 40^\circ = 49$

eqn 1 $\Rightarrow$ $a = \frac{\cos 40^\circ}{\cos 52^\circ} b$

substitute into eqn 2: $\left(\cos 40^\circ \frac{\sin 52^\circ}{\cos 52^\circ} + \sin 40^\circ\right) b = 49$

so $b = \frac{49}{\cos 40^\circ \tan 52^\circ + \sin 40^\circ} \approx 30.19\text{N}$, $a = \frac{\cos 40^\circ}{\cos 52^\circ} b \approx 37.50\text{ N}$

so $\underline{T}_1 = -23.12\,\underline{i} + 29.59\,\underline{j}$, $\underline{T}_2 = 23.12\,\underline{i} + 19.41\,\underline{j}$