HW0 (calculus review)

Due: January 31, Wed., in class

Note: For the first 4 questions of this assignment, you can write down their answers directly (no work needed). For the last question (on integrals), you need to show all your work.

1. Let $A = \{1, 2, 3\}, B = \{3, 4\}$. Find $A \cap B$ and $A \cup B$.

2. Find the domain and range of the function

$$f(x) = \frac{1}{1 + x^2}.$$ 

3. For which values of $p$ is the following series convergent?

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

4. Determine each of the following sums:

$$\sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i} = ?$$

$$\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + \cdots = ? \quad \text{(assuming } |r| < 1\text{)}$$

$$\sum_{n=0}^{\infty} \frac{A^n}{n!} = \frac{1}{0!} + \frac{A}{1!} + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots = ? \quad \text{(} A \text{ is any fixed real number)}$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n + 1)} = \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots = ?$$

5. Evaluate the following integrals

$$\int_1^{\infty} \frac{2}{x^3} \, dx, \quad \int_0^1 x(1 - x)^3 \, dx, \quad \int_0^{\infty} xe^{-2x} \, dx, \quad \int_0^{\infty} xe^{-x^2} \, dx$$