

have always encountered just before running into shock waves in the solar wind upstream of planets. Shortly after the oscillations, Voyager was in the new solar wind regime of heightened magnetic field. Everyone, including Krimigis, now agrees that this new regime is the heliosheath.

Now that they are in it, researchers are eager to understand the heliosheath. They missed recording the actual passage through the shock because it occurred during one of the gaps in Voyager monitoring by the big radio telescopes of the Deep Space Network. But they will be studying the heightened tur-

bulence within the heliosheath and how the turbulence helps deflect galactic cosmic rays. The spacecraft's reports from the heliosheath should also help scientists understand similar shock-bounded "astrospheres" seen around other, more energetic stars.

Researchers are also looking outward toward the next Voyager milestone: leaving the heliosphere entirely. Estimates of the distance to the heliopause—where solar wind ends and the interstellar medium begins—vary widely. Gurnett's interpretation of radio signals emanating from that frontier place it anywhere from 116 AU to 177 AU. But

Voyager 1 will run short of power from its radioisotope thermal generator as early as 2020 and go silent about 147 AU out.

Now, knowing where the termination shock is, researchers are suggesting 125 AU as a best estimate of the distance to the heliopause. "That's a comforting number," says Gurnett, because it would get Voyager 1 there around 2014. Perhaps NASA managers will be equally comforted and remove Voyager 1 and its lagging companion Voyager 2 from the list of space physics missions to be considered this fall for termination.

—RICHARD A. KERR

NUMBER THEORY

Third Time Proves Charm for Prime-Gap Theorem

Dan Goldston feels much better. Two years ago the number theorist at San Jose State University in California suffered a discouraging setback. He and Cem Yildirim of Boğaziçi University in Istanbul, Turkey, had announced a dramatic breakthrough in the theory of prime numbers, only to learn that their proof contained a fatal error (*Science*, 4 April 2003, p. 32; 16 May 2003, p. 1066). But now, with the help of János Pintz of the Alfréd Rényi Mathematical Institute in Budapest, Hungary, Goldston and Yildirim have unveiled a new proof of their breakthrough result. This time experts who have examined it say the proof is rock-solid—in part because it is much simpler than the earlier attempt.

"It's of enormous importance," says Brian Conrey, director of the American Institute of Mathematics in Palo Alto, California. "It's going to open the door to lots of stuff." Andrew Granville of the University of Montreal, Quebec, whose work helped torpedo the original flawed proof, agrees. "It's quite a turning point," he says.

Goldston and Yildirim were studying the way one prime number follows another. Prime numbers—positive integers such as 2, 3, 5, 7, 11, and 13, which can't be broken down into smaller factors—become rarer as numbers get larger. On average, the gap between a large prime p and the next prime number is approximately the natural logarithm of p , written $\log p$. But the actual gap

between two primes may be far from average. Number theorists long ago proved that there is no upper limit on how large the gap can grow, relative to $\log p$. What Goldston and Yildirim claimed—and, together with Pintz, have now proved—is that the smallest possible gap also continues to shrink relative to $\log p$, as the numbers increase.

The original proof foundered when Granville and Kannan Soundararajan of the University of Michigan, Ann Arbor, spotted a mistake in a single, technical subsection of the proof, known as a lemma. The rest of the proof was fine, and part of it immediately enabled two other mathematicians to make a major breakthrough in studying arithmetic progressions of primes (*Science*, 21 May 2004, p. 1095). Goldston and Yildirim also salvaged a weaker result about prime gaps that improved on previous researchers' work.

Goldston kept hoping to make the proof work but finally gave up. "I had come to terms with not getting a good result," he recalls. Then, about a year ago, he had an idea for a new approach. He worked out the details and presented his new proof last summer at the mathematical conference center in Oberwolfach, Germany. He woke up the next morning, however, knowing he had made another mistake, this time in the very last step of the proof. "I really felt jinxed by the whole thing," he recalls.

Pintz, however, took a close look at the flawed proof and came up with the key insight for the ultimate fix. He contacted Goldston and Yildirim last December, and the three number theorists had a complete proof by early February. This time, they were more cautious about announcing the result. "We all thought it was wrong," Goldston says. They circulated the manuscript to a handful of experts, including Granville and Soundararajan, asking them to probe it for any new or remaining errors.

In addition to finding nothing wrong, the ad hoc jury also discovered ways to simplify the proof. "It's been simplified so much there's not much room for an error to be hiding," says Conrey. One of the experts, Yoichi Motohashi of Nihon University in Japan, found a shortcut that led to a surprisingly short proof of the basic, qualitative result. He and the three lead authors have posted this proof, running a mere eight pages, at the arXiv preprint server (www.arxiv.org). The more-detailed paper with Pintz is being rewritten to incorporate some of the simplifications. Goldston gave a public presentation on the new proof at a number theory conference held from 18 to 21 May at the City University of New York.

In itself, the basic result is not a surprise. But it may help mathematicians tackle the famous "twin prime" conjecture, which probably dates back as far as mathematicians have thought about prime numbers. The conjecture holds that there are infinitely many primes for which the gap is 2. The list of twin primes starts with (3, 5), (5, 7), and (11, 13), and has been tabulated by now into the trillions. No one knows whether twin primes ever stop appearing. The new proof is still a far cry from the twin prime conjecture, but it offers a glimmer of hope that number theorists may eventually get there—perhaps a lot sooner than they ever expected. "The twin prime conjecture doesn't seem impossible to prove anymore," Goldston says.

—BARRY CIPRA



Comeback kid. Goldston despaired of rescuing his proof, but a bright idea saved the day.