Answers for some of the homework problems from sections 10.1-10.2

10.1.8. There are $2^n$ Boolean functions of degree $n$, so the answer is $2^{2^7} = 2^{128}$.

10.1.10. One can use a table to solve the problem. Alternatively, one can reason as follows. The only way for the sum to have the value 1 is for one of the summands to have the value 1, since $0 + 0 + 0 = 0$. Each summand is 1 if and only if the two variables in product making up that summand are both 1. The conclusion follows.

10.2.2. The standard way to solve the problem is using a table. We can reason also as follows.

(a) Let’s rewrite $F$ as $F(x, y) = \overline{x} \cdot 1 + \overline{y} \cdot 1 = \overline{x}(y + \overline{y}) + \overline{y}(x + \overline{x})$. Expanding and using that $\overline{x}y + \overline{y}x = \overline{xy}$ we obtain: $\overline{x}y + \overline{y}x + xy$.

(b) This is already in sum-of-products form.

(c) We need to write the sum of all products, i.e. the answer is: $xy + \overline{x}y + \overline{y}x$.

(d) Similar to part (a), we obtain $F(x, y) = 1 \cdot \overline{y} = (x + \overline{x})\overline{y} = xy + \overline{x}y$.

10.2.4.

(a) We need to write all terms that have $\overline{x}$ in them. Thus the answer is $\overline{x}yz + \overline{x}y\bar{z} + \overline{x}\bar{y}z + \overline{x}\bar{y}\bar{z}$.

(b) We need to write all terms that include either $\overline{x}$ or $\overline{y}$. Thus the answer is $x\overline{yz} + x\overline{y}\bar{z} + \overline{x}y\overline{z} + \overline{x}\overline{y}\overline{z} + \overline{x}\bar{y}\bar{z}$.

(c) We need to write all terms that include both $\overline{x}$ and $\overline{y}$. Thus the answer is $\overline{x}\overline{y}z + \overline{x}\overline{y}\bar{z}$.

(d) We need to write all terms that have at least one of the $x$, $y$, and $z$. This is all the terms except $xyz$ and so the answer is $xyz + x\overline{y}z + x\overline{y}\bar{z} + x\overline{y}\bar{z} + \overline{x}y\overline{z} + \overline{x}\overline{y}\bar{z}$.

10.2.12. We apply numerous times DeMorgan’s law to replace the occurrence of $s + t$ by $\overline{st}$.

(a) $(x + y) + z = (\overline{(x + y)}\bar{z}) = \overline{xy} \bar{z}$ (solved in class).

(b) $x + \overline{y}(x + z) = (\overline{(x\overline{y}(x + z))}) = (\overline{x}(\overline{y}(x\bar{z})))$.

(c) $\overline{x}\overline{y} = \overline{xy}$.

(d) The second factor is changed in a manner similar to part (a), and so the answer is $\overline{x}(\overline{xy}z)$.