

Math 108, problem set 01b
Due: Wed Jan 30
Last revision due: Wed Feb 13

Problems to be turned in:

The goal of this problem set is to outline the proof of the following theorem.

Theorem 1. If v_1, v_2, v_3, v_4, v_5 are contained in \mathbf{R}^n (n a natural number), then

$$\text{span}\{v_1, v_2, v_3\} \subseteq \text{span}\{v_1, v_2, v_3, v_4, v_5\}.$$

Here are all of the definitions you need to make sense of the statement of this theorem.

Definition. For a natural number n , $\mathbf{R}^n = \{(x_1, \dots, x_n) \mid x_i \in \mathbf{R}\}$.

Definition. For $c \in \mathbf{R}$, $(x_1, \dots, x_n) \in \mathbf{R}^n$, and $(y_1, \dots, y_n) \in \mathbf{R}^n$, we define

$$\begin{aligned}(x_1, \dots, x_n) + (y_1, \dots, y_n) &= (x_1 + y_1, \dots, x_n + y_n), \\ c(x_1, \dots, x_n) &= (cx_1, \dots, cx_n).\end{aligned}$$

Definition. For a natural number k , and $v_1, \dots, v_k \in \mathbf{R}^n$, we define the *span* of v_1, \dots, v_k to be

$$\text{span}\{v_1, \dots, v_k\} = \{c_1v_1 + \dots + c_kv_k \mid c_i \in \mathbf{R}\}.$$

1. Name one particular element v of \mathbf{R}^5 , and explain why the definition tells you that v is an element of \mathbf{R}^5 .
2. Let v_1, v_2 , and v_3 be vectors in \mathbf{R}^n for some natural number n . Name one particular element w of $\text{span}\{v_1, v_2, v_3\}$, expressed in terms of v_1, v_2 , and v_3 , and explain why the definition tells you that w is an element of $\text{span}\{v_1, v_2, v_3\}$.
3. The inclusion “ $\text{span}\{v_1, v_2, v_3\} \subseteq \text{span}\{v_1, v_2, v_3, v_4, v_5\}$ ” is part of the statement of theorem 1. Express this inclusion as an equivalent if-then statement.
4. Write out assumptions and conclusions for theorem 1, putting as much as is logically possible into the assumptions.
5. Use the definition of the span of a subset of \mathbf{R}^n to rewrite the assumptions and conclusions from problem 4. The result should essentially be one or two lines away from a complete proof.