

Math 108, problem set 03
Outline due: Mon Feb 11
Completed version due: Wed Feb 13
Last revision due: Mon Mar 17

Exercises (to be done but not turned in): 8.3-8.5, 8.8-8.10; 9.2-9.4, 9.8-9.9.

Problems to be turned in: All numbers refer to problems in the Yellow Book.

1. 8.6 (all parts).
2. Let A_i be a set for each $i \in \mathbf{N}$. One of the following statements must then be true, and the other need not be true.

- $\bigcup_{i \in \mathbf{N}} A_i \subseteq \bigcap_{i \in \mathbf{N}} A_i$.
- $\bigcap_{i \in \mathbf{N}} A_i \subseteq \bigcup_{i \in \mathbf{N}} A_i$.

Prove the statement that must always be true, and give a counterexample for the other one. (I.e., name a specific A_i for each $i \in \mathbf{N}$ such that the statement fails to hold.)

3. Let $I = \{x \in \mathbf{R} \mid 0 < x < 1\}$. For $x \in I$, let $A_x = [-2 - x, 2 + x]$, i.e., let A_x be the closed interval between $-2 - x$ and $2 + x$. Prove that

$$\bigcup_{x \in I} A_x = (-3, 3) = \{x \in \mathbf{R} \mid -3 < x < 3\}.$$

4. 8.12.
5. Let A , B , C , and D be sets. Complete and prove the following theorem, adding as weak an extra condition as possible: If $A \times B \subseteq C \times D$ and (extra condition), then $A \subseteq C$.
6. Let X and Y be sets, and let Z be a subset of $X \times Y$. Furthermore, let

$$A = \{x \in X \mid (x, y) \in Z \text{ for some } y \in Y\},$$
$$B = \{y \in Y \mid (x, y) \in Z \text{ for some } x \in X\}.$$

Is it always the case that $Z = A \times B$? Prove or give a specific counterexample (i.e., a specific X , Y , and Z where $Z \neq A \times B$).

7. 9.4.