

Math 108, problem set 04
REVISED THU FEB 21
Outline due: Wed Feb 20
Completed version due: Mon Feb 25
Last revision due: Mon Mar 17

Definitions:

block If $\{A_\alpha \mid \alpha \in I\}$ is a partition of a set X , we call the A_α the **blocks** of the partition.

Exercises (to be done but not turned in): 10.1, 10.3, 11.2, 11.5.

Problems to be turned in: All numbers refer to problems in the Yellow Book.

1. 10.1(i).
2. 10.3.
3. Define a relation \sim on \mathbf{R}^2 by saying that $(x_1, x_2) \sim (y_1, y_2)$ if and only if at least one of $x_1 - y_1$ and $x_2 - y_2$ is an integer. Determine whether or not this relation is reflexive, symmetric, or transitive. If a property holds, prove that it holds; if it does not, prove that it does not. If the relation is an equivalence relation, give the equivalence class of a general point.
4. 10.5.
5. 11.1(i).
6. The set of all straight lines with slope 1 (i.e., $y = x$, $y = x + 1$, etc.) form a partition of \mathbf{R}^2 . Choose an equivalence relation \sim whose equivalence classes are the blocks of this partition, and prove that the equivalence classes of your chosen \sim actually are the straight lines with slope 1.
7. Suppose that X and Y are nonempty disjoint sets, and for two nonempty disjoint sets I and J , suppose that $\{A_\alpha \mid \alpha \in I\}$ and $\{A_\alpha \mid \alpha \in J\}$ are partitions of X and Y , respectively. Prove that $\{A_\alpha \mid \alpha \in I \cup J\}$ is a partition of $X \cup Y$.