

**Math 108, problem set 05**  
**(Due dates revised Tue Feb 26)**  
**Outline due: WED MAR 05**  
**Completed version due: MON MAR 10**  
**Last revision due: Wed Apr 23**

**Exercises (to be done but not turned in):** 12.1, 12.4, 12.5, 12.6, 12.8.

**Problems to be turned in:** All numbers refer to problems in the Yellow Book.

1. Let  $S = \left\{ 2 + \frac{7}{n} \mid n \in \mathbf{Z}^+ \right\}$ . Determine (guess) the value of  $\inf S$ , and prove that your guess is correct.
2. 12.7.
3. Let  $S$  be a nonempty bounded subset of  $\mathbf{R}$ , and let  $x$  be a real number such that  $x > \inf S$ . Prove that there exists some  $s \in S$  such that  $x > s$ . (Suggestion: What if there isn't?)
4. 12.10.
5. 12.12.
6. 12.14.
7. (a) Is it possible that there exists a largest positive real number, i.e., a real number  $M$  such that  $x \leq M$  for all  $x \in \mathbf{R}$ ? Prove or disprove.  
(b) Is it possible that there exists a smallest positive real number, i.e., a real number  $m > 0$  such that  $x \geq m$  for all positive  $x \in \mathbf{R}$ ? Prove or disprove.