

**Math 108, problem set 06**  
**Outline due: Wed Mar 12**  
**Completed version due: Mon Mar 17**  
**Last revision due: Wed Apr 23**

**Exercises (to be done but not turned in):** 13.1, 13.2, 13.3, 13.5, 13.8, 14.4, 14.5, 14.8.

**Problems to be turned in:** All numbers refer to problems in the Yellow Book. You will also need to use the definition of *composite function* from chapter 15 (p. 175).

1. 13.5.
2. Complete (in as interesting a manner as possible) and prove the following theorem: Let  $A$ ,  $B$ , and  $Y$  be sets, and let  $f : A \rightarrow Y$  and  $g : B \rightarrow Y$  be well-defined functions. Then the formula

$$h(x) = \begin{cases} f(x) & \text{if } x \in A, \\ g(x) & \text{if } x \in B, \end{cases}$$

gives a well-defined function  $h : (A \cup B) \rightarrow Y$  if and only if (condition on  $f$  and  $g$ ).

3. 13.12(b,d).
4. Define  $f : \mathbf{R} \rightarrow \mathbf{R}$  by  $f(x) = 7 - (x - 3)^2$ . Find  $\text{ran}(f)$  and carefully prove that your answer is correct (i.e., do a full set equality proof).
5. Let  $a$  be a fixed (constant) real number, and define  $f : \mathbf{R} \rightarrow \mathbf{R}$  by  $f(x) = ax$ .
  - (a) For which values of  $a$  is  $f$  one-to-one? Prove your answer (i.e., prove that  $f$  is one-to-one for suitable values of  $a$  and prove that  $f$  is not one-to-one for other values of  $a$ ).
  - (b) For which values of  $a$  is  $f$  onto? Prove your answer (i.e., prove that  $f$  is onto for suitable values of  $a$  and prove that  $f$  is not onto for other values of  $a$ ).
6. Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be functions. Prove that if  $f$  is one-to-one and  $g$  is one-to-one, then  $g \circ f$  is one-to-one.
7. Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be functions. Prove that if  $f$  is onto and  $g$  is onto, then  $g \circ f$  is onto.