

**Math 108, problem set 07**  
**Outline due: Wed Apr 02**  
**Completed version due: Mon Apr 07**  
**Last revision due: Wed Apr 23**

**Exercises (to be done but not turned in):** 15.3, 15.9, 16.1, 16.2, 16.3, 16.6.

**Problems to be turned in:** All numbers refer to problems in the Yellow Book.

1. 15.4.
2. Let  $f : \mathbf{N} \rightarrow \mathbf{N}$  and  $g : \mathbf{N} \rightarrow \mathbf{N}$  be defined by

$$f(n) = n + 1,$$
$$g(n) = \begin{cases} n - 1 & \text{if } n > 0, \\ 0 & \text{if } n = 0. \end{cases}$$

Is  $f \circ g = \text{id}_{\mathbf{N}}$ ? Is  $g \circ f = \text{id}_{\mathbf{N}}$ ? Are  $f$  and  $g$  inverses? Explain completely, and prove any assertions you make.

3. 15.10.
4. 15.14.
5. Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be defined by  $f(x) = |x|$ .
  - (a) For  $a, b \in \mathbf{R}$  and  $a < b$ , precisely describe the image  $f((a, b))$ , where  $(a, b)$  is the open interval between  $a$  and  $b$ . Note that this description may or may not require several cases, depending on  $a$  and  $b$ .
  - (b) For  $a, b \in \mathbf{R}$  and  $a < b$ , precisely describe the preimage  $f^{-1}((a, b))$ . Note that  $f$  is not invertible; note also that this description may or may not require several cases, depending on  $a$  and  $b$ .
6. 16.11.
7. 16.14.