Math 108, problem set 08
Outline due: Wed Apr 09
Completed version due: Mon Apr 14
Last revision due: Mon May 14

Exercises (to be done but not turned in): 17.3, 17.4, 17.5, 18.3, 18.4, 18.6, 18.7.
Problems to be turned in: All numbers refer to problems in the Yellow Book.

1. 17.9.

2. 17.11.

3. Let $a_1, a_2, \ldots, a_n$ be positive real numbers, with $n \geq 2$. Use induction to prove that

$$a_1^2 + \cdots + a_n^2 < (a_1 + \cdots + a_n)^2.$$

4. In this question, you are to assume that the following theorem is true.

   Theorem. If $f(x)$ is a polynomial with complex coefficients such that $\deg f \geq 1$, then $f(x) = (x - a)q(x)$ for some polynomial $q(x)$ with complex coefficients.

   Use the above theorem to prove the following corollary.

   Corollary. If $f(x)$ is a polynomial with complex coefficients such that $\deg f = n \geq 1$, then $f(x)$ is the product of n polynomials of degree 1.

   (Suggestion: Induction on $n = \deg f$. You may use the fact that, for nonzero polynomials $f(x)$ and $g(x)$, $\deg fg = \deg f + \deg g$.)

5. Recall that an integer $p > 1$ is said to be *prime* if, whenever $p = ab$ for $a, b \in \mathbf{N}$, either $a = 1$ or $b = 1$. Prove the following theorem using strong induction on n:

   Theorem. Any integer $n \geq 2$ is the product of one or more primes.

   (Suggestion for induction step: Either n is prime or it isn't.)

6. 18.2(a). No proofs necessary.

7. 18.3. No proofs necessary, except for last question in (c).