

Math 108, problem set 09
Outline due: Wed Apr 16
Completed version due: Mon Apr 21
Last revision due: Mon May 12

Exercises (to be done but not turned in): 18.8, 18.11, 18.12, 18.14, 19.1, 19.4, 19.5, 19.6.

Problems to be turned in: All numbers refer to problems in the Yellow Book.

1. 18.6.
2. 18.8.
3. Guess the value of $\lim_{n \rightarrow \infty} \frac{1}{n^2 + 2}$, and use the definition of convergence to prove that your guess is correct.
4. Guess the value of $\lim_{n \rightarrow \infty} \frac{2n + 5}{3n - 1}$, and use the definition of convergence to prove that your guess is correct.
5. 19.5.
6. Note that each part of this problem may be useful in the subsequent parts. Also, the point of this problem is to prove Thm. 19.9 part (iii), so please do not use Thm. 19.9, parts (iii) and (iv), in this problem. (Using parts (i) and (ii) is fine.)
 - (a) Let a_n be a sequence, and let L be a real number. Define the sequence $x_n = a_n - L$. Prove that $\lim_{n \rightarrow \infty} a_n = L$ if and only if $\lim_{n \rightarrow \infty} x_n = 0$. (Suggestion: Carefully apply Thm. 19.9, parts (i) and (ii). Note that we can think of x_n as an “error term” indicating how far away a_n is from the limit L ; the result can be interpreted to say that $\lim_{n \rightarrow \infty} a_n = L$ if and only if the error goes to 0 as $n \rightarrow \infty$.)
 - (b) Let x_n and y_n be sequences such that $\lim_{n \rightarrow \infty} x_n = 0$ and $\lim_{n \rightarrow \infty} y_n = 0$. Prove that $\lim_{n \rightarrow \infty} x_n y_n = 0$.
 - (c) Let a_n and b_n be sequences such that $\lim_{n \rightarrow \infty} a_n = L$ and $\lim_{n \rightarrow \infty} b_n = M$. Prove that $\lim_{n \rightarrow \infty} a_n b_n = LM$. (Suggestion: Let $x_n = a_n - L$ and $y_n = b_n - M$, and rewrite $a_n b_n$ in terms of x_n , y_n , L , and M .)