

**Math 108, problem set 10**  
**Outline due: Wed Apr 30**  
**Completed version due: Mon May 05**  
**Last revision due: TBA**

**Exercises (to be done but not turned in):** 20.7, 20.9, 20.13, 21.4, 21.5.

**Problems to be turned in:** All numbers refer to problems in the Yellow Book.

1. 19.12(b).
2. Let  $(x_n)$  be a bounded sequence, and for each positive integer  $n$ , let

$$s_n = \inf \{x_m \mid m \geq n\}.$$

In the following, use the theorems of Problem 19.12.

- (a) Prove that  $(s_n)$  is an increasing sequence.
  - (b) Prove that  $(s_n)$  is a bounded sequence.
  - (c) Prove that  $(s_n)$  converges. (The limit of  $(s_n)$  is called the **limit inferior** of  $(x_n)$ , or the **lim inf** of  $(x_n)$  for short.)
3. 20.5.
  4. 20.7. (Define a function in (a); don't define a function in (b).)
  5. 20.9.
  6. 20.10.
  7. 21.7.