Format and topics
Exam 1, Math 108

General information. Exam 1 will be a timed test of 75 minutes, covering Chapters 1–11 of the Yellow Book, Sections 1–8 of the proof notes, and the reading on "knights and knaves". No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Types of questions. There are four types of questions that may appear on exams in this class, namely:

1. Statements of definitions and theorems;
2. Proofs;
3. Proof outlines;
4. True/false with justification.

Statements of definitions and theorems. In these questions, you will be asked to recite a definition or the statement of a theorem from the book. You will not be asked to recite the proofs of any theorems from the book, though you may be asked to prove book theorems that you might have been asked to prove on problem sets. (Note that in Math 108, we will concentrate almost exclusively on definitions, as for the most part, the definitions in this class are more important than the theorems.)

Proofs. These will resemble some of the shorter problems from your homework. You may take as given anything that has been proven in class, in the homework, or in the reading. Partial credit may be given on proof questions, so keep trying if you get stuck (and you've finished everything else). If all else fails, at least try to write down the definitions of the objects involved (i.e,. supply an outline!).

Proof outlines. In these questions, you will be asked to outline the proof of a theorem, and not finish the proof. In such a question, you need to state the assumptions and conclusions of the theorem clearly and split the proof into parts if the proof has an obvious Part 1/Part 2 structure (e.g., if the theorem is an "if and only if" statement). Furthermore, if the conclusion of the theorem involves a "there exists" statement, you should indicate the point where you have to construct or choose a corresponding object (i.e., "Let $p =$?").

True/false with justification. This type of question (which is not likely to be on Exam 1, but may be on future exams) may be less familiar to you. You are given a statement, such as:

- If A lives on the island of knights and knaves, then A is capable of saying "If I am a knight, then 2+2=5."

If the statement is true, all you have to do is write "True". (However, see below.) If the statement is false (like the one above), not only do you have to write "False", but you must also give a reason why the statement is false. Your reason might be a specific counterexample, like:

> If A is a knave, then A would be saying, "If (false), then (false)", which would mean that A is making a true statement; contradiction. So not everyone is capable of making that statement.

Or your reason might be a very general principle:

> By going through all cases in a truth table (work omitted), we see that it is not possible for *anyone* on the island of knights and knaves to make that statement.

Either way, your answer should be as specific as possible to ensure full credit.

Depending on the problem, some partial credit may be given if you write "False" but provide no justification, or if you write "False" but provide insufficient or incorrect justification. Partial credit may also be given if you write "True" for a false statement, but provide some partially reasonable justification. (In other words, if you have time, it can't hurt to justify "True" answers.)

If I can't tell whether you wrote "True" or "False", you will receive no credit. In particular, please do not just write "T" or "F", as you may not receive any credit.

Definitions. The most important definitions we have covered in the yellow book (*Reading, Writing, and Proving*) are:

Ch. 6	$A \subseteq B$	subset
	contained	proper subset
	equal	$A = B$
	union	$A \cup B$
	intersection	$A \cap B$
	empty set	disjoint
	set difference	$A \backslash B$
	complement A^c	
Ch. 8	index set	indexed family
	union of a family of sets	$\bigcup_{\alpha \in I} A_\alpha$
	intersection of a family of sets	$\bigcap_{\alpha \in I} A_\alpha$
Ch. 9	power set	ordered pair (informal)
	Cartesian product	$X \times Y$
	relation from X to Y	relation on X
Ch. 10	reflexive relation	symmetric relation
	transitive relation	equivalence relation
	equivalence class	
Ch. 11	partition	

You do not need to know definitions from either the proof notes or the *Concepts* book.

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Ch. 8 Examples 8.2, 8.6.
Ch. 9 Example 9.1.
Ch. 10 Example 10.2.

You should also be familiar with all of the examples from the Exercises from Ch. 6–10, and you should be familiar with the examples from PS01a–c and PS02.

Theorems, results, algorithms. The most important theorems, results, algorithms, and axioms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

Ch. 10 Example 10.2.
Ch. 11 Partitions come from equivalence relations, and vice-versa (Thm. 11.4).

Other. Please be familiar with the "techniques of proof" in the proof notes, Sects. 1–8, 9, 12, and 13. You should also be familiar with the ideas, the logic, and the proof techniques from Chs. 1–5 of the Yellow Book.

Not on exam. The material in *Concepts* will not be covered on Exam 1.

Good luck.