

**Format and topics**  
**Exam 3, Math 108**

**General information.** Exam 3 will be a timed test of 50 minutes, covering Chapters 16–21 of the Yellow Book and all of the proof notes. No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we’ve studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should definitely spend time memorizing the *statements* of the important theorems in the text.

**Types of questions.** Exam 3 will feature the same types of questions as Exams 1 and 2.

**Definitions.** The most important definitions we have covered in the Yellow Book are:

Ch. 16	image	inverse image
Ch. 17	base case	induction step
	induction hypothesis	
Ch. 18	sequence	term
	bounded above	bounded below
	bounded	upper bound
	lower bound	inf
	sup	increasing
	decreasing	strictly increasing
	strictly decreasing	recursion
Ch. 19	converges	limit
	diverges	Cauchy sequence
Ch. 20	equivalence (of sets)	same cardinality
	equipotent	equinumerous
	finite	infinite
	restriction (of a function)	

You do not need to know any definitions from the *Concepts* book.

**Examples.** You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

**Ch. 16** Example 16.8.

**Ch. 17** Example 17.2; Exercise 17.5.

**Ch. 18** Exercise 18.8.

**Ch. 19** Examples 19.2, 19.3; bounded does not imply convergent (make your own example); Problem 19.10.

**Ch. 20** Strange examples of sets with the same cardinality (Thm. 20.5, etc.).

**Ch. 21** Examples of provably infinite sets ( $\mathbf{N}$ ,  $\mathbf{Z}$ , etc.)

You should also be familiar with all of the examples from the Exercises from Ch. 16–21, and you should be familiar with the examples from PS07–10.

**Theorems, results, algorithms, axioms.** The most important theorems, results, algorithms, and axioms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don’t have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

**Ch. 16** Properties of images and inverse images (Thms. 16.6–16.7).

**Ch. 17** Principle of mathematical induction.

**Ch. 19** Limits are unique (Thm. 19.7); Convergent implies bounded (Thm. 19.8); Combination theorems (Thm. 19.9).

**Ch. 20** Equivalence/“same cardinality” is an equivalence relation (Thm. 20.1); subset of finite is finite (Cor. 20.11); finite union of finite is finite (Thm. 20.12); finite product of finite is finite (Cor. 20.15).

**Ch. 21** The pigeonhole principle (Thm. 21.2); size of a finite set is well-defined (Thm. 21.6).

**Other.** Please be familiar with the “techniques of proof” in the proof notes.

**Not on exam.** The material in *Concepts* will not be covered on Exam 3. (Ch. 18) Fibonacci sequence, Fibonacci numbers.

**Good luck.**