Sample Exam 1
Math 108, Spring 2008

1. (12 points) Let X be a set, and let $\sim$ be a relation on X. Define what it means for $\sim$ to be an equivalence relation.

2. (14 points) As you know, knights always tell the truth, and knaves always lie. Each of A, B, and C is either a knight or a knave. A and B say:

A: If B is a knight, then I am a knave.
B: C is a knave.

Is A a knight or a knave? Is B a knight or a knave? Is C a knight or a knave? Briefly **EXPLAIN** your answer.

In questions 3–4, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

3. (12 points) Let A and B be sets. If x is not an element of $A \cap B$, then $x \notin A$ and $x \notin B$.

4. (12 points) Let X be a set. If $A \in \mathcal{P}(X)$ (the power set of X), then A is an element of X, i.e., $A \in X$.

5. (16 points) **OUTLINE** the proof of the following theorem. More precisely, state the assumption and the conclusion for the proof of the theorem, and work forwards and backwards towards the middle as much as you can by only applying definitions. **DO NOT TRY TO PROVE THIS THEOREM.**

Theorem. For $n \in \mathbf{Z}^+$ (the set of all positive integers), let

$$A_n = \left\{ x \in \mathbf{R} \mid 0 \leq x \leq 1 - \frac{1}{n} \right\},$$
$$B_n = \left\{ x \in \mathbf{R} \mid 0 \leq x < 1 + \frac{1}{n} \right\}.$$

Then $\displaystyle\bigcup_{n \in \mathbf{Z}^+} A_n \subseteq \bigcap_{k \in \mathbf{Z}^+} B_k$.

6. (16 points) **PROOF QUESTION.** Let A, B and C be sets, and suppose that $A \subseteq B$. Prove that

$$(C \setminus B) \cup ((B \cap C) \setminus A) \subseteq (C \setminus A).$$

7. (18 points) **PROOF QUESTION.** Let

$$A = \left\{ (x, y) \in \mathbf{R}^2 \mid x > 0 \right\},$$
$$B = \left\{ (x, y) \in \mathbf{R}^2 \mid y > 0 \right\},$$
$$C = \left\{ (x, y) \in \mathbf{R}^2 \mid xy > 0 \right\},$$
$$D = \left\{ (x, y) \in \mathbf{R}^2 \mid x + y > 0 \right\}.$$

Prove that $A \cap B = C \cap D$.