Sample Exam 3
Math 108, Spring 2008

1. (14 points) Let (x_n) be a sequence.

(a) Define what it means for (x_n) to be an increasing sequence.

(b) Give an example of a sequence (x_n) that is neither increasing nor decreasing.

In questions 2–4, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

2. (12 points) If A is an infinite set, and B is equivalent to A (i.e., $A \approx B$), then B must be infinite.

3. (12 points) If $f : X \to Y$ is a function, B is a subset of Y, and $A = f^{-1}(B)$, then it must be the case that f is invertible and A is the image of B under f^{-1}. (Recall that f^{-1} denotes the inverse of the function f).

4. (12 points) If A and B are infinite sets, and $A \cap B = \emptyset$, then it is not possible that A is equivalent to $A \cup B$.

5. (16 points) **PROOF QUESTION.** Use the definition of the limit of a sequence to prove that

$$\lim_{n\to\infty} \frac{2n}{3n-1} = \frac{2}{3}.$$

6. (16 points) **PROOF QUESTION.** Let x be a real number such that $1 + x > 0$. Prove by induction that for all positive integers n, $(1 + x)^n \geq 1 + nx$.

7. (18 points) **PROOF QUESTION.** Let (x_n) be a sequence such that $\lim_{n\to\infty} x_n = 0$.

(a) Explain what it means to say that $\lim_{n\to\infty} x_n = 0$, in terms of the definition of limit.

(b) Prove that $\lim_{n\to\infty} |x_n| = 0$.