PS01 outline: answers
Math 126, Fall 2009

An outline for PS01 should look something like the following.

Relevant definitions:

We say that an integer d **divides** an integer n if $n = dq$ for some integer q.

We say that an integer p is **prime** if $p > 1$ and the only (positive) divisors of p are 1 and p.

We say that d is a **common divisor** of a and b if d divides a and d divides b.

For positive integers a and b, the **greatest common divisor** of a and b is the largest integer d that is a common divisor of a and b.

A **triangular number** is a number of the form $\dfrac{n(n+1)}{2}$.

A **Pythagorean triple** is a triple (a, b, c) of natural numbers such that $a^2 + b^2 = c^2$.

A **primitive Pythagorean triple** is a PT (a, b, c) such that a, b, and c have no common factors greater than 1. (I.e., $\gcd(a, b, c) = 1$.)

Outlines of individual problems:

1.1. First try listing all triangular numbers $\dfrac{n(n+1)}{2}$ and see which ones are perfect squares. To look for a better method, maybe think about the prime factorization of $\dfrac{n(n+1)}{2}$, and how that could give a perfect square.

1.3. Try listing all triples of odd numbers, starting with prime numbers: $(3, 5, 7)$, $(5, 7, 9)$, $(7, 9, 11)$, $(11, 13, 15)$, etc., and look for a pattern.

1.4. (a,b,c) Try listing numbers of the form $N^2 - 1$ and see if there seem to be infinitely many primes of that form. Same for $N^2 - 2$, $N^2 - 3$, $N^2 - 4$.

(d) Find a pattern from (a,b,c).

2.2. **Assume:** d divides m and d divides n. **Conclude:** d divides $m + n$ and d divides $m - n$. Use definition of divides.

2.6. (a,b) Trial and error, using PPT formula from theorem on p. 17.

(c) Use PPT formula and solve?

3.3. Imitate proof in Ch. 3: **Assume:** (x, y) is a rational point on $x^2 - y^2 = 1$. Consider line through $(-1, 0)$ and (x, y), with rational slope m, solve for x in terms of m. **Conclude:** $(x, y) = (?, ?)$ (formula involving m).