Math 126, problem set 03
Outline due: Wed Sep 23
Due: Mon Sep 28
Last revision due: Mon Oct 12

1. 7.5.

2. In Theorem 2.1 and its proof, we have checked that if (a, b, c) is a primitive Pythagorean triple, then we can find odd integers $s > t \geq 1$ with $\gcd(s, t) = 1$ such that $a = st$, $b = \dfrac{s^2 - t^2}{2}$, and $c = \dfrac{s^2 + t^2}{2}$. In this problem, you are to prove a converse to this theorem. (See part (b).)

   (a) Prove that (explain why) if $\gcd(a, b) > 1$, then there exists a prime p such that p divides both a and b. Suggestion: Use the Fundamental Theorem of Arithmetic.

   (b) Prove that if $s > t \geq 1$ are odd integers, $\gcd(s, t) = 1$, $a = st$, and $b = \dfrac{s^2 - t^2}{2}$, then $\gcd(a, b) = 1$. Suggestion: Use contradiction, use part (a), and use Claim 7.1 (p. 44).

3. 8.1(b).

4. 8.2.

5. 8.3(a,c).

6. 8.4(b,d).

7. 8.5(b,c).