Sample Exam 1
Math 126, Fall 2009

Note that this sample exam only covers Chs. 1–7, whereas our exam will also cover Ch. 8. In addition, your exam will probably be a bit longer.

1. (12 points) Let a and b be natural numbers. Define what it means for a to divide b.

2. (13 points) State the Fundamental Theorem of Arithmetic. Be brief, but be as precise as possible.

3. (13 points) Let N be a natural number. Describe an efficient and precise (no guessing or trial and error) method for finding a Pythagorean Triple (a, b, c) with $a > N$ and $\gcd(a, b, c) = 1$.

4. (20 points) No explanation necessary, but show all your work.

(a) Find $\gcd(102, 27)$.

(b) Find all solutions to the equation $102x + 27y = \gcd(102, 27)$, $x, y \in \mathbb{Z}$.

For questions 5–6, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer as specifically as possible. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

5. (12 points) Let n, a, and b be natural numbers. If n divides ab, then either n divides a or n divides b.

6. (12 points) Let r be a natural number. If we can find *one* rational point on the circle $x^2 + y^2 = r$, then we can find the coordinates of *all* rational points on $x^2 + y^2 = r$.

7. (18 points) Let a, b, c be natural numbers such that

$$2a^2 + 2b^2 = c^2.$$

Example: $(a, b, c) = (1, 1, 2)$.

Prove that it is **not** possible that 3 divides a and 3 does not divide b (simultaneously). (You may want to try proof by contradiction.)