Sample Exam 2
Math 126, Fall 2009

Again, your exam will probably be a bit longer than this sample (or at least there will be more questions).

1. (10 points) Define the function $\pi(x)$, and state the Prime Number Theorem as precisely as possible.

2. (10 points) Let $m = 2^{10} \cdot 7^3$. Calculate the sum of the divisors of m. No explanation necessary, but show all your work. **DO NOT SIMPLIFY YOUR ANSWER.**

3. (14 points) Consider the congruence

$$6x \equiv 15 \pmod{27}.$$

If this congruence has at least one solution, find a largest possible set of incongruent solutions, showing all your work; if the congruence has no solutions, explain how you can be sure that it has no solutions.

For questions 4–7, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer as specifically as possible. **For full credit, include a counterexample where appropriate.** (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

4. (12 points) Suppose that $m = pqr$, where each of p, q, and r are prime. Then it must be the case that $\varphi(m) = (p-1)(q-1)(r-1)$. ($\varphi$ is Euler's phi function.)

5. (12 points) Let a be a positive integer. Then it must be the case that there exist infinitely many primes congruent to $a \pmod{333}$.

6. (12 points) Let a be an integer such that $\gcd(a, 49) = 1$. Then it must be the case that $a^{48} \equiv 1 \pmod{49}$.

7. (12 points) Let a be an odd integer. Then it must be the case that there exists some integer x such that $ax \equiv 1 \pmod{64}$.

8. (18 points) Note that $275 = 5^2 \cdot 11$. The following statement is always true, given suitable conditions on a:

> Let a be an integer. If (*conditions on a*), then $a^{20} \equiv 1 \pmod{275}$.

(a) Complete the above statement to a true statement, using the loosest possible conditions on a.

(b) Prove your completed statement.

(For partial credit, replace the 20 in a^{20} with a larger number and prove the appropriate statement for that larger number, supplying the appropriate conditions on a.)