Math 128A, problem set 01
Outline due: Wed Jan 28
Due: Mon Feb 02
Last revision due: Mon Feb 16

Problems to be done, but not turned in: (Ch. 0) 1, 3, 11, 13, 15, 41, 49; (Ch. 1) 1, 7, 9, 11, 19.

Fun: (Ch. 0) 14, 52; (Ch. 1) 16.

Problems to be turned in:

1. (Ch. 0) 50.
2. Let t, a, and b be positive integers.
   (a) Prove that if t divides a and t divides b, then t divides $a + b$.
   (b) Without using the Fundamental Theorem of Arithmetic, prove that if t divides a and t divides b, then t divides $\gcd(a, b)$.
3. You are pouring water from a faucet into an empty tank, and you have two containers, one holding 16 liters and the other holding 10 liters. You are allowed to fill either container from the faucet and dump all of it into the tank, and you are allowed to completely fill either container from the tank and dump it. Describe all possible numbers of liters of water you can put into the tank, and prove your answer.
4. (Ch. 0) 38.
5. (a) (Ch. 1) 6.
   (b) Choose two distinct reflections f and g in D_5, and compute fg in your notation.
   (c) (Ch. 1) 8.
   (d) Choose a non-trivial rotation r and a reflection f in D_5, and compute rf.
6. (Ch. 1) 22.
7. This problem concerns the rotational symmetries of an ordinary mattress. If you don't have a mattress handy, you can use your textbook to model these symmetries.
   (a) Describe the rotational symmetries of a rectangular (but non-square) mattress. Construct the corresponding Cayley table.
   (b) The McRoskey Airflex Mattress Company of San Francisco recommends the following procedure for maintaining even wear on a mattress.

   Spring: Rotate the mattress from head to foot.
   Summer: Flip the mattress over from head to foot.
   Fall: Rotate the mattress from head to foot.
   Winter: Flip the mattress over from head to foot.

   Explain why this procedure will ensure that your mattress will wear out as evenly as possible. In particular, explain what "rotate" and "flip" mean in terms of your answer to part (a). (Suggestion: Think of the position of the mattress at the beginning of the year as the identity.)