

**Math 128A, problem set 03**  
**Outline due: Wed Feb 11**  
**Due: Mon Feb 16**  
**Last revision due: Mon Mar 16**

**Problems to be done, but not turned in:** (Ch. 3) 17, 19, 23, 27, 29, 33, 39, 45;  
(Ch. 4) 3, 5, 9, 13.

**Fun:** (Ch. 3) 36, 52.

**Problems to be turned in:**

1. (Ch. 3) 28.
2. (Ch. 3) 32.
3. List all cyclic subgroups of  $D_6$ .
4. Let  $G$  be an abelian group, and let

$$H = \{h \in G \mid h = g^2 \text{ for some } g \in G\}.$$

Prove that  $H$  is a subgroup of  $G$ .

5. Let  $K$  be a subgroup of  $\mathbf{R}^*$  (the multiplicative group of the nonzero reals), and let

$$H = \{A \in GL(3, \mathbf{R}) \mid \det A \in K\}.$$

Prove that  $H$  is a subgroup of  $GL(3, \mathbf{R})$ .

6. Let  $G$  be a group, and let both  $H$  and  $K$  be subgroups of  $G$ . Define

$$HK = \{hk \mid h \in H, k \in K\}.$$

- (a) Give an example to show that  $HK$  may not be a subgroup of  $G$ .
  - (b) Now suppose further that  $hk = kh$  for all  $h \in H$  and  $k \in K$ . Prove that  $HK$  is a subgroup of  $G$ .
7. (a) (Ch. 4) 4.
  - (b) (Ch. 4) 6.