

Math 128A, problem set 05
Outline due: Wed Mar 04
Due: Mon Mar 09
Last revision due: Wed Apr 22

Problems to be done, but not turned in: (Ch. 5) 7, 9, 11, 13, 19, 27, 31, 41, 43;
(Ch. 6) 1, 3, 5.

Fun: (Ch. 5) 52.

Problems to be turned in:

1. (a) (Ch. 5) 38.
(b) Find $\alpha, \beta \in S_4$ such that $|\alpha| = 2$, $|\beta| = 2$, and $|\alpha\beta| = 4$.
(c) Find $\alpha, \beta \in S_n$ such that $|\alpha| = 2$, $|\beta| = 2$, and $|\alpha\beta| = n$. (Suggestion: You may want to consider odd and even n separately.)
2. Let X be a nonempty subset of $\{1, 2, 3, 4, 5, 6\}$, and let

$$H_X = \{\beta \in S_6 \mid \beta(x) \in X \text{ for all } x \in X\}.$$

Prove that H_X is a subgroup of S_6 .

3. (Ch. 5) 12.
4. (Ch. 5) 42.
5. Let $\sigma = (1, 2, 3, 4, 5, 6)$. Pick 3 examples of nontrivial $\tau \in S_6$ and write out both τ and $\sigma\tau\sigma^{-1}$ as a product of disjoint cycles. Can you see a pattern relating the cycle notation for τ to the cycle notation for $\sigma\tau\sigma^{-1}$? Find a pattern that applies uniformly, no matter what τ is, and explain what your pattern is, as precisely as possible. (You do not need to prove that your pattern works; just explain it and give some examples.)
6. (Ch. 6) 6.
7. (Ch. 6) 16.