

Math 128A, problem set 07
Outline due: Wed Mar 18
Due: Mon Mar 31
Last revision due: Wed Apr 22

Problems to be done, but not turned in: (Ch. 7) 19, 23, 25, 27, 29, 31, 39, 43, 47;
(Ch. 8) 1, 5, 11, 15.

Fun: (Ch. 7) 28, 33.

Problems to be turned in:

1. (Ch. 7) 30.
2. (Ch. 7) 34.
3. (Ch. 7) 38.
4. (Ch. 7) 44.
5. Let G be a group of permutations of $\{1, \dots, n\}$, i.e., let G be a subgroup of S_n .
 - (a) We say that G is *transitive* on $\{1, \dots, n\}$ if, for every $a, x \in \{1, \dots, n\}$, there exists some $\sigma \in G$ such that $\sigma(a) = x$. Let $G_1 = \text{stab}_G(1)$. If G is a transitive group of permutations of $\{1, \dots, n\}$, find and prove a formula relating $|G|$ and $|G_1|$.
 - (b) We say that G is *doubly transitive* on $\{1, \dots, n\}$ if, for every $a, b, x, y \in \{1, \dots, n\}$ with $a \neq b$ and $x \neq y$, there exists some $\sigma \in G$ such that $\sigma(a) = x$ and $\sigma(b) = y$. If G is a doubly transitive group of permutations of $\{1, \dots, n\}$, find and prove a formula relating $|G_1|$ (defined in the previous part) and the order of

$$G_{12} = \{\sigma \in G \mid \sigma(1) = 1 \text{ and } \sigma(2) = 2\}.$$

- (c) We say that G is *sharply triply transitive* on $\{1, \dots, n\}$ if, for every $a, b, c, x, y, z \in \{1, \dots, n\}$ with a, b, c distinct and x, y, z distinct, there exists some $\sigma \in G$ such that $\sigma(a) = x$, $\sigma(b) = y$, and $\sigma(c) = z$; and furthermore, the group

$$G_{123} = \{\sigma \in G \mid \sigma(1) = 1, \sigma(2) = 2, \text{ and } \sigma(3) = 3\}$$

is trivial. If G is a sharply triply transitive group of permutations of $\{1, \dots, n\}$, find and prove a formula for $|G|$.

6. (Ch. 8) 8.
7. (Ch. 8) 16.