

**Math 128A, problem set 09**  
**REVISED: Thu Apr 16**  
**Outline due: Wed Apr 15**  
**Due: Mon Apr 20**  
**Last revision due: Wed May 13**

**Problems to be done, but not turned in:** (Ch. 10) 1, 3, 7, 9, 11, 17, 23, 27, 31, 39, 43, 47, 55.

**Fun:** (Ch. 10) 40, 52.

**Problems to be turned in:**

1. Let  $k$  be an odd positive integer, and let  $n = 2k$ . Recall (PS02, PS08, class) that for  $r = R_{360/n}$  and  $f$  any reflection in  $D_n$ , we know that  $r^n = f^2 = e$  and  $frf^{-1} = r^{-1}$ , and that

$$D_n = \{r^i, r^i f \mid 0 \leq i < n\}.$$

Let  $H = \{r^{2i}, r^{2i} f \mid 0 \leq i < k\}$ , and let  $K = \{e, r^k\}$ .

- (a) **DELETE:** Prove that  $H$  and  $K$  are subgroups of  $D_n$ .
- (b) Prove that  $H$  and  $K$  are normal in  $D_n$ .
- (c) Prove that  $D_n \approx H \oplus K$ .

(While you do not need to prove this, it is a fact that  $H \approx D_k$  and  $K \approx Z_2$ , which means that  $D_n \approx D_k \oplus Z_2$ .)

2. (Ch. 10) 14.
3. (Ch. 10) 16.
4. (Ch. 10) 22. You do not need to prove your generalization.
5. Suppose  $\varphi : G \rightarrow D_6$  is a surjective homomorphism and  $|\ker \varphi| = 7$ . Prove that  $G$  has a normal subgroup of order 14 and a non-normal subgroup of order 14.
6. (Ch. 10) 34. **DELETE:** Prove your  $\varphi$  is a homomorphism with the given description.
7. Let  $G$  be an abelian group of odd order whose operation is written multiplicatively. Prove that the function  $\varphi : G \rightarrow G$  defined by  $\varphi(x) = x^2$  is an automorphism of  $G$ . Generalize.