**Format and topics**
**Final exam, Math 128A**

**General information.** The final will be somewhat less than twice as long as our in-class exams, with 135 minutes in which to complete it, and it will take place in our usual room.

The final will be **cumulative**; in other words, the final will cover the topics on this sheet and the topics on the previous three review sheets. However, the exam will somehwat emphasize the material listed here from Chapters 12–14.

As always, most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs may help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

As usual, no books or notes allowed, and four basic types of questions: computations, statements of definitions and theorems, proofs, and true/false with justification.

**Definitions.** The most important definitions we have covered are:

| | | |
|---|---|---|
| Ch. 12 | ring | commutative ring |
| | unity, 1 | unit, $a^{-1}$ |
| | divide, factor, $a \mid b$ | $R[x]$ ($R$ a comm. ring) |
| | $M_n(R)$ ($R$ a comm. ring) | direct sum $R_1 \oplus \cdots \oplus R_n$ |
| | subring | Gaussian integers $\mathbf{Z}[i]$ |
| | $n\mathbf{Z}$ | |
| Ch. 13 | zero-divisor | integral domain |
| | field | characteristic of a ring |
| | char $R$ | |
| Ch. 14 | ideal | proper ideal |
| | $\langle a_1, \ldots, a_n \rangle$ | ideal generated by $a_1, \ldots, a_n$ |
| | factor ring $R/A$ | prime ideal |
| | maximal ideal | |

**Examples.** You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

**Ch. 12** $\mathbf{Z}$, $\mathbf{Q}$, $\mathbf{C}$, $\mathbf{R}$, $\mathbf{Z}_n$, $R[x]$, $M_n(R)$, $n\mathbf{Z}$, real-valued functions, $\mathbf{Z}[i]$. Trivial subring, diagonal matrices, upper triangular matrices.

**Ch. 13** Integral domains: $\mathbf{Z}$, $\mathbf{Z}[i]$, $R[x]$ ($R$ is an integral domain), $\mathbf{Z}_p$. Non-integral domains: $\mathbf{Z}_n$ ($n$ composite), $\mathbf{R}_1 \oplus R_2$. Fields: $\mathbf{Z}_3[i]$, $\mathbf{Q}(\sqrt{2})$.

**Ch. 14** Ideals: $R$ and $\{0\}$ in $R$, $n\mathbf{Z}$ in $\mathbf{Z}$, $\langle a_1, \ldots, a_n \rangle$ in $R$ (examples of this class). Non-ideals: even-degree polynomials in $R[x]$. Factor rings: $\mathbf{Z}/n\mathbf{Z}$, $\mathbf{Z}[i]/\langle 2-i \rangle$, $\mathbf{R}[x]/\langle x^2+1 \rangle$. Prime and maximal ideals of $\mathbf{Z}$, $\mathbf{Z}_{36}$; $\langle x^2+1 \rangle$ maximal in $\mathbf{R}[x]$, $\langle x^2+1 \rangle$ not prime in $\mathbf{Z}_2[x]$.

**Theorems, results, algorithms.** The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

**Ch. 12** Rules of multiplication. Unity and inverses unique. Subring Test.

**Ch. 13** Cancellation. Finite integral domains are fields; $\mathbf{Z}_p$ is a field.

**Ch. 14** Ideal test. Factor rings well-defined. $R/A$ is an integral domain iff $A$ is prime; $\mathbf{R}/A$ is a field iff $A$ is maximal.

**Good luck.**