Sample Exam 1
Math 128A, Spring 2009

This exam covers Chs. 0–3 of Gallian; our exam will also include Ch. 4.

1. Let G be a group, and let H be a nonempty subset of G. State the Subgroup Test for determining if H is a subgroup of G. You may use either the One-Step Test, the Two-Step Test, or the one given in class.

2. Consider the group G given by the following Cayley table:

	a	b	c	x	y	z
a	b	a	y	z	c	x
b	a	b	c	x	y	z
c	x	c	z	y	a	b
x	c	x	a	b	z	y
y	z	y	x	c	b	a
z	y	z	b	a	x	c

(a) What is the identity element of G? Briefly justify your answer.

(b) What is the order of c? Briefly justify your answer, and show all of your work.

For questions 3–6, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

3. The set $H = \{0, 4, 6, 8\}$ is a subgroup of $\mathbf{Z}_{12}$.

4. If n is a positive integer, and $a, b \in D_n$, then it must be the case that $a^{-1}b^{-1}ab = e$.

5. It is possible to find an abelian group G and an element $a \in G$ such that the order of a is infinite.

6. Let S be a set, let $\sim$ be an equivalence relation on S, and let a, b, x, and y be elements of S. It is possible that x is an element of $[a]$ (the equivalence class of a) and also an element of $[b]$, while at the same time, y is an element of $[a]$ but not an element of $[b]$.

7. PROOF QUESTION. Let a and n be positive integers, and let k be an integer. Prove that if $\gcd(a, n)$ divides k, then there exists an integer x such that

$$ax = k \pmod{n}.$$

8. PROOF QUESTION. Let G be a group, and let H and K be subgroups of G. Prove that $H \cap K$ is a subgroup of G.