Sample Exam 2
Math 128A, Spring 2009

Again, since the coverage the last time I taught this class is different, the questions on this sample exam are not completely representative. As always, your best guides for coverage are the homework (PS04–07), the reading (Chs. 4–8), and the review handout.

1. Let G be a group (whose operation is written multiplicatively), let H be a subgroup of G, and let a be an element of G. Define the left coset of H containing a and the right coset of H containing a, making clear which is which.

2. Let

$$\alpha = (1\ 5\ 3)(2\ 7)(4\ 8\ 6), \qquad \beta = (2\ 6)(3\ 4\ 9\ 5)(7\ 8).$$

Compute the permutation $\alpha\beta$, and find the order of $\alpha\beta$. No explanation necessary, but show all your work.

For questions 3–6, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

3. If G and H are finite groups, and $|G| = |H|$, then G and H must be isomorphic.

4. The group $\mathbf{Z}_{45}$ has exactly 5 subgroups.

5. It is possible that there exists a subgroup H of a group G such that $|G| = 100,000$ and $|H| = 26$.

6. If α, β, and γ are elements of S_{12}, α is an even permutation, and β is an odd permutation, then it is possible that $\alpha^3\beta\gamma^{-2}$ is an even permutation.

7. PROOF QUESTION. Let H be a proper subgroup of S_4, and suppose that H contains $(12)(34)$ and (123). Prove that $H = A_4$.

8. PROOF QUESTION. Let a and b be elements of an **abelian** group G such that the order of a is 24, the order of b is 10, and $\langle a \rangle \cap \langle b \rangle = e$. Prove that G contains an element of order 120 and an element of order 30.