Sample Exam 3
Math 128A, Spring 2009

Again, since the coverage the last time I taught this class is different, the questions on this sample exam are not completely representative. As always, your best guides for coverage are the homework (PS07–10), the homework answers (esp. for PS07), the reading (Chs. 8–12), and the review handout.

1. Let R and $\overline{R}$ be rings. Define the direct sum $R \oplus \overline{R}$, making sure to define the elements of the ring, the ring operation, the zero element, and negatives in the ring.

2. List all of the abelian groups of order $2500 = 2^2 5^4$, up to isomorphism. No explanation needed, but show all your work. (Note that $5^3 = 125$ and $5^4 = 625$.)

For questions 3–6, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

3. There are only finitely many different homomorphisms $\varphi : \mathbf{Z} \to \mathbf{Z}$.

4. The group $\mathbf{Z}_4 \oplus \mathbf{Z}_6$ contains exactly one element of order 2.

5. There exists a group G and a homomorphism $\varphi : \mathbf{Z} \to G$ such that $\mathbf{Z}/\ker\varphi \approx \mathbf{Z}_6$.

6. If H is a **cyclic** subgroup of a group G, then it must be the case that H is normal in G.

7. PROOF QUESTION. Define a function

$$\varphi : \mathbf{Z} \oplus \mathbf{Z} \to \mathbf{Z}$$
$$\varphi(a, b) = a + b.$$

Note that if $a, b \in \mathbf{Z}$, then $a + b \in \mathbf{Z}$, which means that φ is well-defined.

(a) Prove that φ is a homomorphism.

(b) Prove that $\ker\varphi$ is cyclic.

8. PROOF QUESTION. Let N be a normal subgroup of a group G, and suppose that $|G| = 56 = 2^3 7$ and $|N| = 8 = 2^3$. Prove that there exists an element $a \in G$ such that every coset of N has the form $a^k N$ for some integer k. (Suggestion: Consider the factor group G/N.)