Math 128b, problem set 03
Due: Fri Feb 17

Problems to be done, but not turned in: (Ch. 15) 51, 53, 55, 57, 59, 61, 63; (Ch. 16) 5, 7, 11, 13, 17 (assumes $\deg 0 = -\infty$), 21, 27, 29, 31, 39, 43.

Fun: (Ch. 16) 22.

Problems to be turned in:

1. (Ch. 15) 58.
2. (Ch. 15) 60.
3. (Ch. 16) 12.
4. (Ch. 16) 20.
5. (Ch. 16) 32.
6. (Ch. 16) 42.
7. Let F be a field, let $M_n(F)$ denote the ring of $n \times n$ matrices with entries in F, and let A be an element of $M_n(F)$.
   (a) Note that if $f(x) \in F[x]$, then $f(A)$ is an element of $M_n(F)$. Restate this phenomenon in terms of a homomorphism.
   (b) Assume that there exists a nonzero polyonmial $p(x)$ such that $p(A) = 0$ (the zero matrix). (It can be shown that this is always the case.) Prove that there exists a unique monic polynomial $m(x)$ of lowest possible degree such that $m(A) = 0$.
   (c) We define the *minimal polynomial* of A to be the monic polynomial $m(x)$ of lowest possible degree such that $m(A) = 0$ (the zero matrix). Prove that if $f(A) = 0$, then $m(x)$ divides $f(x)$.