Math 128b, problem set 04
Due: Fri Feb 24

Problems to be done, but not turned in: (Ch. 17) 1, 3, 5, 9, 11, 15, 17, 23, 25, 27.

Problems to be turned in:

1. Let p be a fixed prime, let $f(x)$ be a polynomial with integer coefficients, and let $\tilde{f}(x)$ be the reduction of $f(x)$ in $\mathbf{Z}_p[x]$.
   (a) Consider the following statements.
      i. $f(x)$ is reducible over $\mathbf{Z}$.
      ii. $f(x)$ is reducible over $\mathbf{Q}$.
      iii. $\tilde{f}(x)$ is reducible over $\mathbf{Z}_p[x]$.

      There are six possible implications between two statements of (i)–(iii). For each possible implication ((i) implies (ii), (ii) implies (i), etc.), either give a proof, give a reference to a result proven in the text, or give a counterexample.
   (b) If we assume that $f(x)$ is monic, what changes in part (a)? Explain.
2. (a) List all monic irreducible polynomials in $\mathbf{Z}_5[x]$ of degree at most 2.
   (b) List all monic irreducible polynomials in $\mathbf{Z}_3[x]$ of degree at most 3.
   (c) List all monic irreducible polynomials in $\mathbf{Z}_2[x]$ of degree at most 4.
3. Find nonconstant polynomials $f(x) \in \mathbf{Z}[x]$ and $g(x), h(x) \in \mathbf{Q}[x]$ such that $f(x) = g(x)h(x)$, but $g(x) \notin \mathbf{Z}[x]$ and $h(x) \notin \mathbf{Z}[x]$. Why doesn't this contradict Thm. 17.2?
4. (Ch. 17) 10.
5. (Ch. 17) 16.
6. (Ch. 17) 18.
7. (Ch. 17) 24.