Math 128b, problem set 06
Due: Fri Mar 10

Problems to be done, but not turned in: (Ch. 18) 33, 35.
Important terms and symbols:

maxprin We say that a proper ideal $\langle a\rangle$ of a commutative ring R is **maxprin** if, whenever $\langle b\rangle$ is a principal ideal of R and $\langle a\rangle \subseteq \langle b\rangle \subseteq R$, then either $\langle b\rangle = \langle a\rangle$ or $\langle b\rangle = R$. In other words, we do not require that $\langle a\rangle$ is maximal among *all* ideals, merely among principal ones.

Problems to be turned in:

1. (Ch. 17) 32.
2. Let D be an integral domain, and let a be an element of D.
   (a) Prove that $\langle a\rangle$ is prime if and only if a is prime.
   (b) Prove that $\langle a\rangle$ is maxprin (see above) if and only if a is irreducible.
   (c) Find an example of a principal idea $\langle a\rangle$ of an integral domain D such that $\langle a\rangle$ is maxprin, but not maximal, in D.
3. Let D be an integral domain such that every $a \in D$ can be factored into irreducible elements of D. Prove that the following conditions are equivalent:
   - D is a unique factorization domain.
   - Every irreducible element of D is prime.
   - Every maxprin ideal of D is a prime ideal.
4. Let D be an integral domain, and let a be an element of D.
   (a) Prove that if $\langle a\rangle$ maximal, then $\langle a\rangle$ is prime. Prove that if $\langle a\rangle$ is prime, then $\langle a\rangle$ is maxprin. Give counterexamples for each converse implication.
   (b) Now suppose that D is a unique factorization domain. Must either of the converse implications in part (a) hold? Prove or give counterexamples.
   (c) Now suppose that D is a principal ideal domain. Must either of the converse implications in part (a) hold? Prove or give counterexamples.
5. (Ch. 18) 30.
6. (Ch. 18) 32.
7. (Ch. 18) 34.