A **vector space** is a set V with

- addition ($\mathbf{v} + \mathbf{w} \in V$ defn for $\mathbf{v}, \mathbf{w} \in V$) and
- scalar multiplication ($r\mathbf{v} \in V$ defn for $r \in \mathbb{R}$, $\mathbf{v} \in V$)

such that for all $\mathbf{v}, \mathbf{w}, \mathbf{x} \in V$, $r, s \in \mathbb{R}$:

1. $\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v}$. (commutativity)
2. $(\mathbf{v} + \mathbf{w}) + \mathbf{x} = \mathbf{v} + (\mathbf{w} + \mathbf{x})$. (associativity)
3. There is a vector in V, called $\mathbf{0}$, such that $\mathbf{v} + \mathbf{0} = \mathbf{v}$. (additive identity)
4. For each $\mathbf{v} \in V$, there exists some $-\mathbf{v} \in V$ such that $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$. (additive inverse)
5. $r(\mathbf{v} + \mathbf{w}) = r\mathbf{v} + r\mathbf{w}$. (distributivity)
6. $(r + s)\mathbf{v} = r\mathbf{v} + s\mathbf{v}$. (distributivity)
7. $r(s\mathbf{v}) = (rs)\mathbf{v}$. (associativity of scalar multiplication)
8. $1\mathbf{v} = \mathbf{v}$. (scalar identity)