Math 129b, problem set 00 (review)
Due: Fri Aug 27
Last revision due: Wed Sep 15

1. Compute $\left(\begin{bmatrix}-1 & 2\\ 0 & 3\end{bmatrix} + 2\begin{bmatrix}1 & 1\\ -2 & -4\end{bmatrix}\right)^T \begin{bmatrix}2\\ 3\end{bmatrix}$.

2. Let $A = \begin{bmatrix}3 & -6 & -2\\ 1 & -3 & 0\\ -2 & 1 & 3\end{bmatrix}$ and let $B = \begin{bmatrix}1 & 0\\ -5 & 7\\ 0 & 3\end{bmatrix}$.

   (a) Choose either AB or BA, as long as the product you choose is defined, and compute its value. Show all your work.

   (b) If A is invertible, compute A^{-1}; if A is not invertible, explain how you know it is not invertible. Show all your work.

3. Let A be a 3×4 matrix whose columns are $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, and let $\mathbf{x} = \begin{bmatrix}w\\ x\\ y\\ z\end{bmatrix}$ be a vector in $\mathbb{R}^4$. In one sentence, describe the vector $A\mathbf{x}$ as completely as possible.

4. Find the general solution of the following system of linear equations, and put your final answer in vector form.

$$\begin{aligned} x_1 + x_2 + x_3 - x_4 &= 0,\\ 2x_2 - 2x_3 - 2x_4 &= -6,\\ -2x_1 - x_2 - 3x_3 \quad &= -7. \end{aligned}$$

5. Given the following matrix A and the reduced row-echelon form of A:

$$A = \begin{bmatrix}1 & 1 & 3 & 5 & -12\\ 2 & 0 & 4 & 1 & -1\\ 3 & -1 & 5 & 1 & -2\end{bmatrix} \qquad \operatorname{rref}(A) = \begin{bmatrix}1 & 0 & 2 & 0 & 1\\ 0 & 1 & 1 & 0 & 2\\ 0 & 0 & 0 & 1 & -3\end{bmatrix}.$$

   (a) Let V be the column space of A. Find a basis for V, and find the dimension of V. No explanation necessary, but show all your work.

   (b) Let W be the null space of A. Find a basis for W, and find the dimension of W. No explanation necessary, but show all your work.

6. Let A be a 4×4 matrix. Explain, in one sentence, what $\det A$ tells you about A^{-1}.

Questions 7–8: Indicate true/false, and justify your answer.

7. If $W = \operatorname{span}\left\{\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}, \begin{bmatrix}2\\ -1\\ 0\end{bmatrix}\right\}$, then W contains exactly two vectors.

8. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be vectors in $\mathbb{R}^3$ such that none of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ is a scalar multiple of another, i.e., $\mathbf{v}_1 \neq c\mathbf{v}_2$ for any $c \in \mathbb{R}$, $\mathbf{v}_1 \neq c\mathbf{v}_3$ for any $c \in \mathbb{R}$, and so on. Then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ must be linearly independent.