Math 129b, problem set 10
Outline due: Wed Nov 24
Due: Wed Dec 01
Last revision due: Mon Dec 08

Problems to be done, but not turned in: (4.1) 4, 8, 10, 15, 17, 21; (4.2) 3, 7, 9, 11, 15, 17; (4.3) 3, 5, 7, 9, 11

Problems to be turned in:

1. Does
$$\langle (v_1, v_2), (w_1, w_2) \rangle = v_1 w_1 - v_1 w_2 - v_2 w_1 + v_2 w_2$$
define an inner product on $\mathbb{R}^2$? Prove or give a counterexample for each axiom that fails.

2. Does $\langle f, g \rangle = \displaystyle\int_0^1 x^2 f(x) g(x)\, dx$ define an inner product on $\mathbb{C}([0,1])$? Prove or give a counterexample for each axiom that fails.

3. (4.1) 22.

4. A *regular n-simplex* S is a set S of $n+1$ points (the *vertices* of the n-simplex S) such that the distance between any two of the points in S is the same. For example, a regular 2-simplex is precisely an equilateral triangle, and a regular 3-simplex is precisely a regular tetrahedron. The *center* of a regular n-simplex S is defined to be the mean of the vertices of S or in other words, the sum of the coordinates of all vertices of S, divided by $n+1$.

   The goal of this problem is to examine the geometry of the regular n-simplex.

   (a) Prove that the points $(-2,1,1)$, $(1,-2,1)$, and $(1,1,-2)$ are the vertices of an equilateral triangle with center $(0,0,0)$.

   (b) Prove that the points $(-3,1,1,1)$, $(1,-3,1,1)$, $(1,1,-3,1)$, and $(1,1,1,-3)$ are the vertices of a regular tetrahedron with center $(0,0,0,0)$.

   (c) Generalize the results of (a) and (b) to find coordinates for the vertices of a regular n-simplex with center $\mathbf{0}$.

   (d) Let $\mathbf{x}$ and $\mathbf{y}$ be vectors from the center of a regular n-simplex to two of its vertices, and let θ_n be the angle between $\mathbf{x}$ and $\mathbf{y}$. Find a formula for $\cos\theta_n$.

   (e) What happens to θ_n as $n \to \infty$? Explain your answer.

5. (4.3) 6.