Format and topics for exam 2
Math 129b

General information. Exam 2 will cover 3.1–3.6, 6.1–6.2, and 6.6 of the text. The exam will be cumulative only to the extent that 3.1–3.6, 6.1–6.2, and 6.6 rely on previous material; for example, you should still know what a subspace is and how to use the Subspace Theorem. However, there will not be any questions on the exam that only cover old material; for example, you will not be asked to recite the Subspace Theorem.

As before, most of the exam will rely on understanding the problem sets (including the problems to be done but not written up or turned in) and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Exam 2 will follows the same ground rules as exam 1 did. In particular, no books, notes, or calculators are allowed, and there will be the same four types of questions: computations, statements of definitions and theorems, proofs, and true/false with justification.

Definitions. The most important definitions we have covered are:

3.1	linear combination	coefficients (of a lin comb)
3.2	span $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$	span (noun)
	to span (verb)	
3.3	linearly independent	linearly dependent
3.4	basis	dimension
	n-dimensional	finite dimensional
	infinite dimensional	
3.5	proof by contradiction	
3.6	coordinate vector	coordinates relative to the basis B
	$[\mathbf{v}]_B$	
6.1	$T : V \to W$	linear
	additivity	homogeneity
	linear map	linear transformation
	linear operator	μ_A
	C_B	L_B
6.2	one-to-one	onto
	composition	identity
	inverse	image
6.6	kernel	null space
	column space	row space

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. Most of the important examples we have encountered have appeared in the assigned problems, both those to be turned in and those not to be turned in. In addition, you should also know:

3.5: Standard bases for $\mathbb{R}^n$, $\mathbb{P}_n$, and $\mathbb{M}(m, n)$.
6.1: Differentiation; integration; matrix multiplication map $\mu_A : \mathbb{R}^n \to \mathbb{R}^m$; coordinate function $C_B : V \to \mathbb{R}^n$; linear combination function $L_B : \mathbb{R}^n \to V$; polynomial substitution $p(x) \to p(x+1)$, etc.; identity map; zero map.
6.2: Differentiation, integration.
6.6: Kernel and image of $\mu_A : \mathbb{R}^n \to \mathbb{R}^m$, differentiation, integration.

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

3.1: Using row-reduction to determine if $\mathbf{v}$ is a linear combination of $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$.
3.2: Span is a subspace; using row-reduction to determine if $\mathbf{v}$ is in span $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$; how to prove that $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ spans a vector space.
3.3: How to prove that a set of vectors is linearly independent; $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ linearly dependent iff one vector is a linear combination of the others.
3.4: $\dim \mathbb{R}^n = n$; $\dim \mathbb{P}_n = n + 1$; $\dim \mathbb{M}(m, n) = mn$.
3.5: Comparison Theorem and corollary (dimension well-defined); Contraction Theorem; Contraction Algorithm; Expansion Lemma; Expansion Theorem; using the Expansion Lemma to prove linear independence; Subspace Size Theorem (Thm. 3.14); The Two Out of Three Theorem (PS05 and Thm. 3.15).
3.6: Coordinates relative to a basis exist and are unique.
6.1: Basic properties of linear maps (Thm. 6.2); any linear map is determined by image of a spanning set.
6.2: Invertible iff one-to-one and onto; composition of linear maps is linear; inverse of (invertible) linear is linear; Whatever Theorem.
6.6: Kernel and image are subspaces; $\operatorname{im} \mu_A$ is column space of A; row operations do not change row space; dim(row space) equals dim(col space); finding bases for kernel and image of $\mu_A : \mathbb{R}^n \to \mathbb{R}^m$; finding bases for row space and column space of $A \in \mathbb{M}(m, n)$.

Good luck.